DOCUMENT RESUME

ED 393 866 TM 024 713

AUTHOR Kim, Seock-Ho; Cohen, Allan S.
TITLE On the Behrens-Fisher Problem: A Review.
PUB DATE 16 Jun 95
NOTE 40p.; Paper presented at the Annual Meeting of the Psychometric Society (Minneapolis, MN, June 1995).
PUB TYPE Reports - Evaluative/Feasibility (142) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS *Bayesian Statistics; *Hypothesis Testing; Probability; *Statistical Inference; Statistical Significance
IDENTIFIERS *Behrens Fisher Problem; P Values; *Variance (Statistical)

ABSTRACT

The Behrens-Fisher problem arises when one seeks to make inferences about the means of two normal populations without assuming the variances are equal. This paper presents a review of fundamental concepts and applications used to address the Behrens-Fisher problem under fiducial, Bayesian, and frequentist approaches. Methods of approximations to the Behrens-Fisher distribution and a simple Bayesian framework for hypothesis testing are also discussed. Finally, a discussion is provided for the use of generalized "p" values in significance testing of hypotheses in the presence of nuisance parameters. It is shown that the generalized "p" value based on a frequentist probability for the Behrens-Fisher problem is numerically the same as those from the fiducial and Bayesian solutions. A table of tests of significance is also included. (Contains 3 tables, 2 figures, and 93 references.) (Author)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
☑ This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.
• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
SEOCK-HO KIM
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)"

ED 393 866

On the Behrens-Fisher Problem: A Review

Seock-Ho Kim
The University of Georgia
and
Allan S. Cohen
University of Wisconsin–Madison

June 16, 1995

Running Head: BEHRENS-FISHER PROBLEM

Paper presented a the annual meeting
of the Psychometric Society, Minneapolis, MN

TM024713

BEST COPY AVAILABLE

On the Behrens-Fisher Problem: A Review

Abstract

The Behrens-Fisher problem arises when one seeks to make inferences about the means of two normal populations without assuming the variances are equal. This paper presents a review of fundamental concepts and applications used to address the Behrens-Fisher problem under fiducial, Bayesian, and frequentist approaches. Methods of approximations to the Behrens-Fisher distribution and a simple Bayesian framework for hypothesis testing are also discussed. Finally, a discussion is provided for the use of generalized p values in significance testing of hypotheses in the presence of nuisance parameters. It is shown that the generalized p value based on a frequentist probability for the Behrens-Fisher problem is numerically the same as those from the fiducial and Bayesian solutions. A table for tests of significance is also included.

Key words: Behrens-Fisher problem, two means problem, unequal variances.

Introduction

A frequently encountered problem in applied statistics is testing the difference between two population means. The Behrens-Fisher problem arises when one seeks to make inferences about the means of two normal populations without assuming either that the variances are equal or that the ratio of variances is known. Under such conditions, Neyman-Pearson (1928, 1933) sampling theory may provide a different solution from those available via either Bayesian theory (e.g., Jeffreys, 1940) or Fisher's (1935, 1939) fiducial theory (Kendall & Stuart, 1979; Lehmann, 1993). Although a number of methods have been proposed for the Behrens-Fisher problem beginning with Behrens (1929) and Fisher (1935), no definitive solutions exist (Robinson, 1982). For reasonably large sample sizes, differences between various extant solutions are generally much less than between these solutions and use of Student's t test. When sample sizes are small, however, the three theories may yield different solutions. In the context of Bayesian and fiducial theories, several sets of tables (Fisher & Healy, 1956; Fisher & Yates, 1957; Issacs, Christ, Novick, & Jackson, 1974; Lindley & Scott, 1984; Sukhatme, 1938; Sukhatme, Thawani, Pendharkar, & Natu, 1951) have been presented for the Behrens-Fisher problem. No tables are available, however, for directional hypothesis testing for even numbers of degrees of freedom and small sample sizes.

This paper presents a review of the Behrens-Fisher problem, focusing on fundamental concepts and applications rather than theoretical and philosophical considerations. It begins with expositions of Fisher's fiducial and Jeffreys' Bayesian approaches to the Behrens-Fisher problem. A table is then presented for significance tests which includes cases for even numbers of degrees of freedom and small sample sizes under directional hypothesis testing. Also discussed are methods of approximations of the Behrens-Fisher distribution and a simple Bayesian framework for hypothesis testing (Lindley, 1965). In addition, frequentist approaches (Aspin, 1948; Tsui & Weerahandi, 1989; Welch, 1938, 1947) are discussed. Finally, it is shown that the generalized p value (Tsui & Weerahandi, 1989) is numerically the same as those obtained from the fiducial and Bayesian solutions. Examples are presented to illuminate similarities and differences among various methods.

Formulation of the Problem

We motivate the discussion of the Behrens-Fisher problem with an example taken from Marascuilo and Serlin (1988, p. 229). In this problem, patients in a mental health clinic were given one of two initial treatments: $n_1 = 4$ patients received a film treatment and $n_2 = 3$ patients received an interview treatment. Researchers wanted to know whether the patients in the film treatment and the interview treatment differed in the number of times they returned to the clinic for subsequent treatment. The following are the data for this problem: $x_1 = (x_{11}, \ldots, x_{1n_1}) = (8, 10, 12, 15)$, $\bar{x}_1 = 11.25$, $s_1^2 = 8.91667$, $x_2 = (1, 7, 11)$, $\bar{x}_2 = 6.33333$, and $s_2^2 = 25.33333$, where the sample mean and sample variance for $i = 1, 2$ are defined as

$$\bar{x}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} x_{ij} \tag{1}$$

and

$$s_i^2 = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2. \tag{2}$$

We assume that the two independent samples, x_{1j} and x_{2j}, were drawn from two normal distributions having means μ_1 and μ_2 and variances σ_1^2 and σ_2^2, respectively. With the assumption of equal variances, that is, $\sigma_1^2 = \sigma_2^2 = \sigma^2$, the population variance is estimated by the pooled sample variance, $s^2 = 15.48333$, using

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}. \tag{3}$$

The sufficient statistics for μ_1, μ_2, and σ^2 are $\bar{x}_1$, $\bar{x}_2$, and s^2. Note also that $\bar{x}_2 - \bar{x}_1$ has a normal distribution with mean $\delta = \mu_2 - \mu_1$ and variance $(1/n_1 + 1/n_2)\sigma^2$. The Student's t pivotal statistic with $n_1 + n_2 - 2$ degrees of freedom is

$$t = \frac{\delta - (\bar{x}_2 - \bar{x}_1)}{\sqrt{(1/n_1 + 1/n_2)s^2}} \sim t(n_1 + n_2 - 2). \tag{4}$$

If we denote $t_{\alpha/2}(\nu)$ as the value for which

$$\Pr\{t > t_{\alpha/2}(\nu)\} = \alpha/2 \tag{5}$$

and also denote

$$\alpha = \Pr\{t < -t_{\alpha/2}(\nu)\} + \Pr\{t > t_{\alpha/2}(\nu)\}, \tag{6}$$

then the $100(1-\alpha)\%$ confidence interval for δ is

$$\bar{x}_2 - \bar{x}_1 \pm t_{\alpha/2}(n_1 + n_2 - 2)\sqrt{(1/n_1 + 1/n_2)s^2}. \tag{7}$$

For the null hypothesis H_0: $\delta = 0$, we have $t = 1.63599$ with 5 degrees of freedom. The resulting two-tailed p value is .16277 with a 95% confidence interval of $[-12.64209, 2.80875]$ or $-12.64209 \leq \delta \leq 2.80875$. The result is that the difference between the two means is not significant at the .05 alpha level.

When it is not reasonable to assume $\sigma_1^2 = \sigma_2^2$, neither a pivotal statistic nor an exact confidence interval procedure exist. One simple way to solve this problem, however, is to use a proxy

$$t^* = \frac{\delta - (\bar{x}_2 - \bar{x}_1)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} \sim t[\min(\nu_1, \nu_2)], \tag{8}$$

where $\nu_1 = n_1 - 1$ and $\nu_2 = n_2 - 1$. Restate in terms of a confidence interval, we have

$$\bar{x}_2 - \bar{x}_1 \pm t_{\alpha/2}[\min(\nu_1, \nu_2)]\sqrt{s_1^2/n_1 + s_2^2/n_2}. \tag{9}$$

Note, in this case (i.e., t^* test), the actual confidence is greater than $100(1-\alpha)\%$ (Mickey & Brown, 1966). The Marascuilo and Serlin (1988) data yield $t^* = 1.50493$; with 2 degrees of freedom, the resulting two-tailed p value is .27127. The 95% confidence interval is $[-18.97365, 9.14031]$. When $n_1 = n_2 = n$, the value obtained from Equation 4 is the same as that from Equation 8. In this special case we can use Equations 4 and 7 to test the null hypothesis and to obtain the $100(1-\alpha)\%$ confidence interval, respectively. However, as noted in Hsu (1938) and Robinson (1976), the Type I error rate might be greater than the specified nominal level unless the equality of two variances is satisfied.

The above t^* solution is a simple ad hoc approach to the Behrens-Fisher problem. Two other approaches based on the test statistic t^* are derived from fiducial theory and Bayesian theory. We first present a solution from fiducial theory followed by a solution based on the Bayesian approach.

Fisher's Fiducial Approach

Fisher (1935) proposed a statistical method for obtaining a probability distribution of a parameter from observed data called a fiducial probability distribution (see Fraser, 1978 for

a brief review). Recall that we assume the two sets of observations are random samples drawn from independent normal distributions. The quantities $\bar{x}_i$ and s_i^2 are jointly sufficient for μ_i and σ_i^2 having independent sampling distributions $N(\mu_i, \sigma_i^2/n_i)$ and $(\sigma_i^2/\nu_i)\chi^2_{\nu_i}$, respectively, for $i = 1, 2$. Consequently, we can define

$$t_i = \frac{\mu_i - \bar{x}_i}{\sqrt{s_i^2/n_i}} \sim t(n_i - 1). \tag{10}$$

By logical inversion

$$\mu_i = \bar{x}_i + t_i\sqrt{s_i^2/n_i} \tag{11}$$

and $\delta = \mu_2 - \mu_1$ is distributed as

$$\delta = \bar{x}_2 - \bar{x}_1 + t_2\sqrt{s_2^2/n_2} - t_1\sqrt{s_1^2/n_1}. \tag{12}$$

The fiducial distribution of δ can be used to make fiducial inferences about δ and to set fiducial intervals. Instead of obtaining the distribution of δ, for the purpose of tabulation, however, Fisher (1935, 1939) chose the statistic

$$\tau = \frac{\delta - (\bar{x}_2 - \bar{x}_1)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = t_2 \cos\theta - t_1 \sin\theta, \tag{13}$$

where θ is taken in the first quadrant and

$$\tan\theta = \frac{s_1/\sqrt{n_1}}{s_2/\sqrt{n_2}}. \tag{14}$$

Noted that τ is the same quantity as t^* but for ease of presentation in the context of both fiducial and Bayesian theories we use τ. The distribution of τ is the Behrens-Fisher distribution and is defined by the three parameters ν_1, ν_2, and θ, $\tau = \tau(\nu_1, \nu_2, \theta)$. The distribution can be seen as a mixture of two t distributions, $t(\nu_1)$ and $t(\nu_2)$.

If we denote $\tau_{\alpha/2}(\nu_1, \nu_2, \theta)$ as the value for which

$$\Pr\{\tau > \tau_{\alpha/2}(\nu_1, \nu_2, \theta)\} = \alpha/2, \tag{15}$$

then the $100(1 - \alpha)\%$ fiducial interval is

$$\bar{x}_2 - \bar{x}_1 \pm \tau_{\alpha/2}(\nu_1, \nu_2, \theta)\sqrt{s_1^2/n_1 + s_2^2/n_2}. \tag{16}$$

Bayesian Approach

In this section, we present the Bayesian approach to the Behrens-Fisher problem given by Jeffreys (1940, 1961). This solution is equivalent to that obtained via the fiducial inference approach. From the fundamental theorem of the normal distribution, the quantities $\bar{x}_i$ and s_i^2 are jointly sufficient for μ_i and σ_i^2, which have independent sampling distributions $N(\mu_i, \sigma_i^2/n_i)$ and $(\sigma_i^2/\nu_i)\chi^2_{\nu_i}$ for $i = 1, 2$. Then, assuming vague reference prior distributions for μ_1, μ_2, $\log\sigma_1$, and $\log\sigma_2$ (i.e., independent and locally uniform), it can be shown that the joint posterior distribution of μ_1 and μ_2 is

$$p(\mu_1, \mu_2|x) = p(\mu_1|\bar{x}_1, s_1^2)p(\mu_2|\bar{x}_2, s_2^2), \tag{17}$$

where $x = (x_1, x_2)$ and for $i = 1, 2$,

$$p(\mu_i|\bar{x}_i, s_i^2) = \frac{(s_i/\sqrt{n_i})^{-1}}{B(\frac{1}{2}, \frac{\nu_i}{2})\sqrt{\nu_i}}\left[1 + \frac{n_i(\mu_i - \bar{x}_i)^2}{\nu_i s_i^2}\right]^{-(\nu_i+1)/2}, \quad -\infty < \mu_i < \infty, \tag{18}$$

where $B(p, q)$ is the beta function, $B(p, q) = \Gamma(p)\Gamma(q)/\Gamma(p+q)$. Hence, a posteriori μ_1 and μ_2 are independent and distributed as scaled t distributions, $t(\bar{x}_1, s_1^2/n_1, \nu_1)$ and $t(\bar{x}_2, s_2^2/n_2, \nu_2)$, respectively.

In order to obtain the posterior distribution of $\delta = \mu_2 - \mu_1$, we transform $\delta = \mu_2 - \mu_1$ and $\delta_1 = \mu_1$ and then integrate out δ_1 from the joint distribution of δ and δ_1:

$$p(\delta|x) = \int p(\delta, \delta_1|x)d\delta_1, \quad -\infty < \delta < \infty. \tag{19}$$

Since the transformation has unit Jacobian, it follows that

$$p(\delta|\bar{x}_1, s_1^2, \bar{x}_2, s_2^2) = k\int_{-\infty}^{\infty}\left[1 + \frac{n_1(\delta_1 - \bar{x}_1)^2}{\nu_1 s_1^2}\right]^{-(\nu_1+1)/2}\left[1 + \frac{n_2(\delta + \delta_1 - \bar{x}_2)^2}{\nu_2 s_2^2}\right]^{-(\nu_2+1)/2} d\delta_1, \tag{20}$$

where

$$k = \frac{(s_1/\sqrt{n_1})^{-1}(s_2/\sqrt{n_2})^{-1}}{B(\frac{1}{2}, \frac{\nu_1}{2})\sqrt{\nu_1}B(\frac{1}{2}, \frac{\nu_2}{2})\sqrt{\nu_2}}. \tag{21}$$

The distribution of $\delta = \mu_2 - \mu_1$ can be computed by numerical integration to any reasonable level of accuracy. We present some approximations for the distribution of δ later for cases where ν_1 and ν_2 are relatively large.

For convenience, we use $\tau = t_2 \cos\theta - t_1 \sin\theta$, where, in terms of the scaled t distribution, t_1 and t_2 are independently distributed as $t(0, 1, \nu_1)$ and $t(0, 1, \nu_2)$, respectively. The posterior distribution of $\mu_2 - \mu_1$ and the inference about δ can be equivalently determined from the distribution of τ. The density function of τ may be obtained from the joint distribution of t_1 and t_2 by the following integration:

$$p(\tau|\nu_1, \nu_2, \theta) = \int\int_{\tau = t_2 \cos\theta - t_1 \sin\theta} p(t_1)p(t_2)dt_1 dt_2, \quad -\infty < \tau < \infty, \tag{22}$$

where

$$p(t_i) = \frac{1}{B(\frac{1}{2}, \frac{\nu_i}{2})\sqrt{\nu_i}} \left(1 + \frac{t_i^2}{\nu_i}\right)^{-(\nu_i+1)/2}, \quad i = 1, 2. \tag{23}$$

To integrate, we set $\tau = t_2 \cos\theta + t_1 \sin\theta$ and $\zeta = t_2 \sin\theta + t_1 \cos\theta$. Since the resulting transformation has unit Jacobian,

$$p(\tau|\nu_1, \nu_2, \theta) = k' \int_{-\infty}^{\infty} \left[1 + \frac{(\zeta \cos\theta - \tau \sin\theta)^2}{\nu_1}\right]^{-(\nu_1+1)/2} \left[1 + \frac{(\zeta \sin\theta + \tau \cos\theta)^2}{\nu_2}\right]^{-(\nu_2+1)/2} d\zeta, \tag{24}$$

where

$$k' = \frac{1}{B(\frac{1}{2}, \frac{\nu_1}{2})\sqrt{\nu_1} B(\frac{1}{2}, \frac{\nu_2}{2})\sqrt{\nu_2}}. \tag{25}$$

The distribution of τ depends only on three parameters ν_1, ν_2, and θ, a result identical to the case given by Fisher (1935, 1939) using fiducial theory. The distribution of τ (i.e., Behrens-Fisher distribution) is a symmetric distribution, similar to the t distribution in appearance. Since Bayesian highest posterior density (HPD) intervals are numerically identical with Fisher's fiducial intervals, they can be obtained using tables such as Sukhatme (1938).

Tables of the Behrens-Fisher Distribution

A number of tables have been presented for the Behrens-Fisher distribution beginning with Sukhatme (1938) and Sukhatme et al. (1951). Subsequently, Fisher and Yates (1957, Tables 6 and 6-1) presented significance points of the Behrens-Fisher distribution for $\nu_1, \nu_2 = 6, 8, 12, 24, \infty$, $\theta = 0°(15°)90°$, $\alpha = .05, .01$ and $\nu_1 \geq \nu_2 = 1(2)7$, $\theta = 0°(15°)90°$, $\alpha = .10, .05, .02, .01$. Weir (1966) presented percentage points of the Behrens-Fisher distribution for $\nu_1, \nu_2 = 6, 8, 12, 24, \infty$, $\theta = 0°(15°)90°$, $\alpha = .001$. Table 41 in Issacs et al.

(1974) contains Behrens-Fisher percentage points for $\nu_1, \nu_2 = 6, 8, 12, 24, \infty$, $\theta = 0°(15°)45°$, $\alpha = .50, .20, .10, .05, .02$. Lindley and Scott (1984, Tables 11-a and 11-b) provide tables for the Behrens-Fisher distribution for $\nu_1 \geq \nu_2 = 1(1)8, 10, 12, 24, \infty$, $\theta = 0°(15°)90°$, $\alpha = .05, .01$. Tables of percentage points of the Behrens-Fisher distribution are also reproduced in Finney (1952), Lee (1989), and Novick and Jackson (1974) (see also Greenwood & Hartley, 1962; Johnson, Kotz, & Balakrishnan, 1995). No tables are yet available, however, when the degrees of freedom for ν_1 and ν_2 are small and even for $\alpha = .10, .02$.

The purpose of this section is to explain the method of approximation used in the computer program for the percentage points of the Behrens-Fisher distribution and to present a table of percentage points for $\nu_1, \nu_2 = 1(1)8, 10, 12, 24, \infty$, $\theta = 0°(15°)90°$, $\alpha = .10, .05, .02, .01$. The method is based on Sukhatme (1938). The complete tables of the Behrens-Fisher distribution for $\nu_1, \nu_2 = 1(1)30, \infty$, $\theta = 0°(15°)90°$, $\alpha = .10, .05, .02, .01$ are available from the authors. In addition, two computer programs, B and BF, which can be used to obtain tail areas and percentage values of the Behrens-Fisher distribution are available from the authors.

The probability that τ exceeds τ_0 can be written as

$$\Pr\{\tau > \tau_0\} = \int_{\tau_0}^{\infty} p(\tau|\nu_1, \nu_2, \theta) d\tau, \tag{26}$$

where $p(\tau|\nu_1, \nu_2, \theta)$ is given in Equation 24. Transforming $t_1 = \zeta \cos\theta - \tau \sin\theta$ and $t_2 = \zeta \sin\theta + \tau \cos\theta$, and changing the order of integration, it follows that

$$\Pr\{\tau > \tau_0\} = \int_{-\infty}^{\infty} p(t_1) \int_{t_0}^{\infty} p(t_2) dt_2 dt_1, \tag{27}$$

where $t_0 = \tau_0 / \cos\theta + t_1 \tan\theta$ and $p(t_i)$ is defined in Equation 23. Next, if we define

$$f(t_1) = p(t_1) \int_{t_0}^{\infty} p(t_2) dt_2 = p(t_1) \Pr\{t_2 > t_0\}, \tag{28}$$

then

$$\Pr\{\tau > \tau_0\} = \int_{-\infty}^{\infty} f(t_1) dt_1 = \lim_{n \to \infty} \sum_{k=1}^{n} f(c_k) \Delta t_1 \approx \sum_{k=1}^{n} f(c_k) \Delta t_1. \tag{29}$$

As $c \to \pm\infty$, $f(c) \to 0$ and f is a nonnegative continuous function. The integral can be approximated via numerical integration to any reasonable degree of accuracy. In the computer programs we set $\Delta t_1 = 10^{-1}$ and define c_1 as the largest number with one decimal

place, where both $c_1 < 0$ and $f(c_1)\Delta t_1 \geq 10^{-15}$ [i.e., $f(c_1) \geq 10^{-14}$], and c_n as the smallest number with one decimal place where both $c_n > 0$ and $f(c_n)\Delta t_1 \geq 10^{-15}$ [i.e., $f(c_n) \geq 10^{-14}$].

Following Box and Tiao (1973), the integration process can be viewed geometrically in Figure 1. In the figure, the joint distribution of t_1 and t_2, [i.e., the product $p(t_1)p(t_2)$], is illustrated by the three dimensional plot. Also shown is the distribution of τ. For a given τ_0, the equation $\tau_0 = t_2 \cos\theta - t_1 \sin\theta$ determines a line on the two dimensional t_1t_2-plane. $\Pr\{\tau > \tau_0\}$ can be obtained by aggregating $\Pr\{t_2 > t_0\}$ weighted by $p(t_1)$.

Insert Figure 1 about here

To illustrate the method, let us suppose that we want to calculate the tail area of $\tau_0 = 1.99704$ for $\nu_1 = \nu_2 = 60$ and $\theta = 45°$. The computer program B first calculates c_1 using temporary values of t_1. If $t_1 = -10.4$, then $f(-10.4) < 10^{-15}$ and, consequently, $c_1 = -10.3$. Similarly with $\Delta t_1 = 10^{-1}$, we obtain $c_n = 4.5$. Values of t_1, values of t_0 corresponding to $\tau_0 = 1.99704$, $\Pr\{t_2 > t_0\}$, and values of $f(c_k)\Delta t_1$ are given in Table 1. The probability that τ exceeds τ_0 is .025. Executing B requires input values for τ_0, ν_1, ν_2, θ.

Insert Table 1 about here

The computer program BF can be used to obtain percentage values of τ. To obtain percentage points, the program requires input for ν_1, ν_2, θ, α, where

$$\alpha = \Pr\{\tau < -\tau_{\alpha/2}(\nu_1, \nu_2, \theta)\} + \Pr\{\tau > \tau_{\alpha/2}(\nu_1, \nu_2, \theta)\}. \tag{30}$$

The $100(1-\alpha)\%$ HPD interval, which is in fact the same as the fiducial interval, can be constructed from a single run of the computer program BF. The $100(1-\alpha)\%$ HPD interval for δ is given in Equation 16. Table 2 contains the percentage points of the Behrens-Fisher distribution for $\nu_1, \nu_2 = 1(1)8, 10, 12, 24, \infty$, $\theta = 0°(15°)90°$, $\alpha = .10, .05, .02, .01$.

Insert Table 2 about here

For the Marascuilo and Serlin (1988) data, $\tau_0 = 1.50493$, $\nu_1 = 3$, $\nu_2 = 2$, $\theta = 27.19365$. From the computer program B, we obtain $\Pr\{\tau > \tau_0\} = .14546$, indicating the difference between the two means is not significant at $\alpha = .05$. From the computer program BF, we obtain $\tau_{.025}(3, 2, 27.19365) = 4.13069$. The corresponding 95% HPD interval is [−18.41183, 8.57850].

Approximation of the Posterior Distribution

Since the Behrens-Fisher distribution has no simple form, approximations have been used to obtain percentage points (e.g., Barnard, 1984; Banerjee, 1960, 1961; Cochran, 1964; Cochran & Cox, 1950; Fisher, 1941; Ghosh, 1975; Linssen, 1991; Patil, 1965; Rahman & Saleh, 1974). For example, Banerjee (1960, 1961) proposed the approximation

$$\tau^{\mathrm{B}}_{\alpha/2} = \sqrt{t^2_{\alpha/2}(\nu_1)c + t^2_{\alpha/2}(\nu_2)(1-c)}, \tag{31}$$

where

$$c = \frac{s_1^2/n_1}{s_1^2/n_1 + s_2^2/n_2} = \sin^2\theta \tag{32}$$

and, consequently,

$$1 - c = \frac{s_2^2/n_2}{s_1^2/n_1 + s_2^2/n_2} = \cos^2\theta. \tag{33}$$

Cochran and Cox (1950) proposed

$$\tau^{\mathrm{CC}}_{\alpha/2} = t_{\alpha/2}(\nu_1)c + t_{\alpha/2}(\nu_2)(1-c). \tag{34}$$

Reporting the statement of a p value, whether or not it is followed by a pronouncement concerning the significance of the result, may sometimes be more informative than reporting a fixed-level hypothesis testing result. In the context of Bayesian inference, it is desirable to show the whole posterior distribution rather than particular intervals. The posterior distribution of δ can be obtained by numerical integration of Equation 19 or using formulas presented by Ghosh (1975). All methods for the approximations of the percentage points of the Behrens-Fisher distribution can be used to plot the approximate posterior distribution.

One important point may be that neither the numerical integration using Equation 19 nor Ghosh's method expresses the posterior distribution in terms of tabled functions. To do

so, we can use Patil's (1965) approximation or Molenaar's (1979) methods. For example, Patil's (1965) method fits the t distribution to the distribution of τ by equating the second and the fourth moments. Patil noted that the formula is valid for $\nu_1, \nu_2 \geq 5$ and works quite well for $\nu_1, \nu_2 \geq 7$. It can be shown that τ is approximately distributed as

$$\tau = \frac{t(g)}{h}, \tag{35}$$

where

$$g = 4 + \left(\frac{\nu_2 \cos^2\theta}{\nu_2 - 2} + \frac{\nu_1 \sin^2\theta}{\nu_1 - 2} \right)^2 \left[\frac{\nu_2^2 \cos^4\theta}{(\nu_2 - 2)^2(\nu_2 - 4)} + \frac{\nu_1^2 \sin^4\theta}{(\nu_1 - 2)^2(\nu_1 - 4)} \right]^{-1} \tag{36}$$

and

$$h^2 = \frac{g}{g-2} \left(\frac{\nu_2 \cos^2\theta}{\nu_2 - 2} + \frac{\nu_1 \sin^2\theta}{\nu_1 - 2} \right)^{-1}. \tag{37}$$

In terms of the scaled t distribution, τ is approximately distributed as $t(0, h^{-2}, g)$. Consequently, the difference of the mean values $\delta = \mu_2 - \mu_1$ is distributed a posteriori as $t[\bar{x}_2 - \bar{x}_1, (s_1^2/n_1 + s_2^2/n_2)/h^2, g]$. Patil's approximation (or Molenaar's methods) cannot be used on the data from Marascuilo and Serlin (1988) due to the small degrees of freedom. We can use Ghosh's (1975) method, however, although neither approximation is better than the numerical integration method. The posterior distribution of δ obtained from the numerical integration method is presented in Figure 2. The posterior distribution of δ, $p(\delta|\mathbf{x})$, is centered at $\bar{x}_2 - \bar{x}_1 = 4.91667$. The 95% HPD interval $[-18.41183, 8.57850]$ is also presented in Figure 2.

Insert Figure 2 about here

Hypothesis Testing Based on Lindley

Lindley (1965) suggested a simple test of a point null hypothesis, for example, H_0: $\delta = \delta_0$ against H_1: $\delta \neq \delta_0$ under a vague or diffuse prior. To conduct a significance test at level α, Lindley suggests finding a $100(1-\alpha)\%$ HPD region. H_0 is rejected only if δ_0 is outside of this region. Notice that a directional hypothesis test can also be easily accommodated. Berger (1985) and Lee (1989) contain other Bayesian approaches to hypothesis testing.

In the Bayesian context, it is often more useful to obtain the whole posterior distribution. The 95% HPD interval in Figure 2 contains 0 and the null hypothesis H_0: $\delta = 0$ cannot be rejected.

Frequentist Approach

Other solutions to the Behrens-Fisher problem also exist (e.g., Aspin, 1948, 1949; Gronow, 1951; James, 1959; Lee & Gurland, 1975; Mehta & Srinivasan, 1970; Nel, van der Merwe, & Moser, 1990; Pagurova, 1968; Scheffé, 1943; Sprott & Farewell, 1993; Tsui & Weerahandi, 1989; Wald, 1955; Welch, 1938, 1947). Several solutions to the Behrens-Fisher problem have been investigated on the basis of their size and magnitude of their power (e.g., Davenport & Webster, 1975; Lee & Gurland, 1975; Mehta & Srinivasan, 1970; Wang, 1971). Of all those methods, perhaps those due to Welch (1938, 1947), Aspin (1948, 1949), and Tsui and Weerahandi (1989) are the most important from the frequentist perspective.

Welch (1938) presented an approximation of t^* by the method of moments and later (Welch, 1947) used asymptotic series expansions to obtain a critical value for terms of order $1/\nu_i^2$:

$$t^{\mathrm{W}}_{\alpha/2}(\nu_1, \nu_2, c) = z_{\alpha/2}\left(1 + V_1 + V_2\right), \tag{38}$$

where $z_{\alpha/2}$ is the $\alpha/2$ point of the standard normal distribution (i.e., the probability of exceeding $z_{\alpha/2}$ is $\alpha/2$) and

$$V_1 = \left(\frac{1 + z^2_{\alpha/2}}{4}\right) V_{21}, \tag{39}$$

and

$$V_2 = -\left(\frac{1 + z^2_{\alpha/2}}{2}\right) V_{22} + \left(\frac{3 + 5z^2_{\alpha/2} + z^4_{\alpha/2}}{3}\right) V_{32} - \left(\frac{15 + 32z^2_{\alpha/2} + 9z^4_{\alpha/2}}{32}\right) V^2_{21}, \tag{40}$$

where c is given in Equation 32 and

$$V_{ru} = \frac{c^r}{\nu_1^u} + \frac{(1-c)^r}{\nu_2^u}. \tag{41}$$

The result was extended further by Aspin (1948) to terms of order $1/\nu_i^3$ and $1/\nu_i^4$ and the latter has become known as the Welch-Aspin test:

$$t^{\mathrm{WA}}_{\alpha/2}(\nu_1, \nu_2, c) = z_{\alpha/2}\left(1 + V_1 + V_2 + V_3 + V_4\right), \tag{42}$$

where V_3 and V_4 are defined in Aspin's (1948, p. 90) Equations 13 and 14, respectively. Lee and Gurland (1975) noted that Aspin's Equation 14 contained an error: Specifically, that V_{42} should be V_{22}. The Welch-Aspin test rejects the null hypothesis H_0 at level α when $t^* > t^{WA}_{\alpha/2}(\nu_1, \nu_2, c)$. We can also obtain the $100(1-\alpha)\%$ confidence interval as

$$\bar{x}_2 - \bar{x}_1 \pm t^{WA}_{\alpha/2}(\nu_1, \nu_2, c)\sqrt{s_1^2/n_1 + s_2^2/n_2}. \tag{43}$$

Aspin (1949) presented tables of critical values of the Welch-Aspin test for $\nu_1, \nu_2 = 6, 8, 10, 15, 20, \infty$, $c = 0(.1)1$, $\alpha = .10$ and $\nu_1, \nu_2 = 10, 12, 15, 20, 30, \infty$, $c = 0(.1)1$, $\alpha = .02$. Trickett, Welch, and James (1956) presented tables for $\nu_1, \nu_2 = 8, 10, 12, 15, 20, \infty$, $c = 0(.1)1$, $\alpha = .05$ and $\nu_1, \nu_2 = 10, 12, 15, 20, 30, \infty$, $c = 0(.1)1$, $\alpha = .01$. Pearson and Hartley (1970, Table 11) reproduced Aspin (1949) and Trickett et al. (1956). Note that critical values for certain small sample sizes (e.g., $\nu_i < 6$) are not available from these tables.

In addition Welch (1938) suggested another method based on a random number of degrees of freedom. This is often referred to as the approximate degrees of freedom solution or Welch's approximate t test. In this regard Welch (1938) presented the following critical value:

$$t^{Wt}_{\alpha/2} = t_{\alpha/2}\left[\nu' = \frac{1}{c^2/\nu_1 + (1-c)^2/\nu_2}\right], \tag{44}$$

where the degrees freedom ν' is in general not an integer. The critical value $t^{\nu'}_{\alpha/2}$ can be obtained using the computer program ET (Galen Research, 1992). When using the t table it is customary to round ν' down to the nearest integer although the impact of such rounding has not been investigated. It is of interest to note that the equivalent of Welch's approximate t test had been proposed by Smith (1936) in a somewhat different context for a related problem (see also Cochran, 1964; Satterthwaite, 1941, 1946; Wallace, 1980). Wang (1971) indicated that Welch's approximate t test is not as effective as the Welch-Aspin test. Even so Best and Rayner (1987), Davenport and Webster (1975), and Scheffé (1970) indicated that Welch's approximate t test may be the best practical solution.

For the Marascuilo and Serlin (1988) data $t^* = 15.0493$ and the critical values are $t^{W}_{.025} = 3.20703$ and $t^{Wt}_{.025} = t_{.025}(3.05344) = 3.15117$. The corresponding 95% confidence intervals of the Welch test of order $1/\nu_i^2$ and Welch's approximate t test are $[-15.39420, 5.56086]$ and $[-15.21169, 5.37835]$, respectively.

For the significance testing of one-sided hypotheses, for example, H_0: $\delta \leq \delta_0$ versus H_1: $\delta > \delta_0$, Tsui and Weerahandi (1989) suggested use of generalized extreme regions of the form

$$R_x(\delta, \eta) = \{X : T(X; x, \delta, \eta) \geq T(x; x, \delta, \eta)\}, \tag{45}$$

where x denotes the observed data for the random variable X, $T(X; x, \delta, \eta)$ is a test statistic that is stochastically increasing in δ, and η is the nuisance parameter. The generalized p-value is defined as

$$p = \sup_{\delta \leq \delta_0} \Pr\{X \in R_x(\delta, \eta)\} \tag{46}$$

and it is free of the nuisance parameter η. A small value of p suggests that the observed x does not support H_0.

In the context of the Behrens-Fisher problem we use the following notations of random variables as $X_i = (X_{i1}, \ldots, X_{in_i})$, $\bar{X}_i \sim N(\mu_i, \sigma_i^2/n_i)$, and $\nu_i S_i^2/\sigma_i^2 \sim \chi^2(\nu_i)$, for $i = 1, 2$. Define x_i, $\bar{x}_i$, and s_i^2 as the observed values of X_i, $\bar{X}_i$ and S_i^2, respectively. Note that the parameter of interest is $\delta = \mu_2 - \mu_1$ and that the nuisance parameter η contains both σ_1^2 and σ_2^2. In place of $T(X; x, \delta, \eta)$, Tsui and Weerahandi (1989) considered a generalized test variable

$$W(X) = W(X_1, X_2; x_1, x_2, \sigma_1^2, \sigma_2^2) = (\bar{X}_2 - \bar{X}_1)\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)^{-1/2}\left(\frac{\sigma_1^2 s_1^2}{n_1 S_1^2} + \frac{\sigma_2^2 s_2^2}{n_2 S_2^2}\right)^{1/2} \tag{47}$$

and hence

$$W(x) = W(x_1, x_2; x_1, x_2, \sigma_1^2, \sigma_2^2) = \bar{x}_2 - \bar{x}_1. \tag{48}$$

The distribution of W given x_1, and x_2 is free of the nuisance parameter and can be seen as $Z(s_1^2/U + s_2^2/V)^{1/2}$, where Z, U, and V are independent, $Z \sim N(\delta, 1)$, $U \sim \chi^2(\nu_1)$, and $V \sim \chi^2(\nu_2)$. In addition, the family of cumulative distributions of $W(X)$ for given x_1 and x_2 is stochastically increasing in δ (Lehmann, 1986). The generalized extreme regions can be defined as

$$R_{x_1,x_2}(\delta, \sigma_1^2, \sigma_2^2) = \{(X_1, X_2) : W(X) - W(x) \geq 0\} \tag{49}$$

and the generalized p value is

$$p = \Pr\{W(X) \geq W(x) | \delta = 0\} = \Pr\left\{Y(\nu_1 + \nu_2)^{-1/2}\left(\frac{\nu_1 s_1^2}{n_1 C} + \frac{\nu_2 s_2^2}{n_2(1-C)}\right)^{1/2} \geq \bar{x}_2 - \bar{x}_1\right\} \tag{50}$$

$$= E\left\{\Psi\left[(\bar{x}_1 - \bar{x}_2)\left(\frac{\nu_1 s_1^2}{n_1 C} + \frac{\nu_2 s_2^2}{n_2(1-C)}\right)^{-1/2}(\nu_1+\nu_2)^{1/2}\right]\right\}, \tag{51}$$

where Y and C are independent, $Y = Z\left[(U+V)/(\nu_1+\nu_2)\right]^{-1/2}$ has a form of Student's t distribution with $\nu_1+\nu_2$ degrees of freedom, $C = U/(U+V) \sim \text{beta}(\nu_1/2, \nu_2/2)$, and $\Psi(\cdot)$ is the cumulative density function of Student's t distribution with $\nu_1+\nu_2$ degrees of freedom. The expectation E is performed with respect to C. Equation 51 can be written as

$$\int_0^1 \Pr\left\{t(\nu_1+\nu_2) \le \tau_0\phi(C)|C\right\} \frac{1}{B(\frac{\nu_1}{2}, \frac{\nu_2}{2})} C^{(\nu_1/2)-1}(1-C)^{(\nu_2/2)-1}dC, \tag{52}$$

where

$$\tau_0 = \frac{(x_1 - x_2)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} \tag{53}$$

and

$$\phi(C) = \left[\frac{(\nu_1+\nu_2)C(1-C)}{\nu_1(1-C)\sin^2\theta + \nu_1 C\cos^2\theta}\right]^{1/2}. \tag{54}$$

Equation 52 is, in fact, the same as $\Pr\{\tau \le \tau_0\} = 1 - \Pr\{\tau > \tau_0\}$, where $\Pr\{\tau > \tau_0\}$ is defined in Equation 26 (cf. Ruben,1960). For H_0: $\delta \le 0$ with $\delta = \mu_2 - \mu_1$, we obtained $\Pr\{\tau \le \tau_0\} = .85454$ from the computer program B. The data do not support the null hypothesis. If we use a different δ (e.g., $\delta = \mu_1 - \mu_2$), however, we have

$$p = \int_0^1 \Pr\left\{t(\nu_1+\nu_2) \ge \tau_0\phi(C)|C\right\} \frac{1}{B(\frac{\nu_1}{2}, \frac{\nu_2}{2})} C^{(\nu_1/2)-1}(1-C)^{(\nu_2/2)-1}dC \tag{55}$$

and Equation 55 and Equation 26 are numerically identical.

Summary

The Behrens-Fisher problem has been one of the focal points of the controversy between the Neyman-Pearson frequentist and Fisherian approaches (as well as possibly the Bayesian approach) to significance testing and statistical inference (for further discussion, see Bartlett, 1936; Fisher, 1956; Lehmann, 1993; Wallace, 1980; Welch, 1956). Although substantial agreement can be found among the results from the various methods, there is also substantial disagreement among proponents of various positions.

Table 3 summarizes the critical values and the 95% confidence/HPD intervals for the data from Marascuilo and Serlin (1988) used to illustrate these methods. The conservative

t^* test yielded the largest critical value and, consequently, the widest confidence interval. The t test yielded the smallest critical value and the shortest confidence interval. All other confidence/HPD intervals lie between the two intervals (see also Mickey & Brown, 1966). Under the assumption that $\sigma_1^2 = \sigma_2^2$, the confidence interval of Equation 7, which is a frequentist solution, coincides with the fiducial interval and as well as with a Bayesian HPD interval based on the vague reference prior. For directional hypothesis testing, the frequentist solution by Tsui and Weerahandi's (1988) yields the same conclusion as fiducial and Bayesian approaches.

Insert Table 3 about here

From the Neyman-Pearson approach, efforts continue to construct or approximate a similar test for which the control of Type I error holds for all σ_i^2, $i = 1, 2$. Linnik (1968/1966) proved, however, that a uniformly most powerful test does not exit in the context of the Behrens-Fisher problem. As indicated in Lehmann (1993) and Wallace (1980), the main difference between the Bayesian (or Fisher) and Neyman-Pearson approaches is whether or not the inference is conditioned.

Bayesian inference is conditional depending on unknown nuisance parameters. The Bayesian context discussed in this paper was only for the noninformative prior. In the general Bayesian context in whcih informative priors may be used, the attempt is made to incorporate the prior information. If one has substantial prior information about the parameters, especially information which can be approximated by independent normal and chi-square distributions for μ_i and σ_i^2, respectively, then the Behrens-Fisher problem can be extended to incorporate it. Other Bayesian methods for the Behrens-Fisher problem have also been presented by Broemeling, Son, and Hamdy (1990), Dayal and Dickey (1976), Johnson and Weerahandi (1988), and Patil (1964).

When there are more than two groups, the Behrens-Fisher problem becomes the generalized Behrens-Fisher problem. When more than one variable is of interest, the Behrens-Fisher problem becomes a multivariate one. Procedures for the generalized Behrens-Fisher problem have been proposed by James (1951), Johansen (1980), Welch (1951), Tamhane (1977), and Tsakok (1978). Comparisons of the solutions and the corresponding robustness

problem for general linear hypotheses is treated by Brown and Forsythe (1974) and Clinch and Kesselman (1982). The multivariate Behrens-Fisher problem in which two groups and two or more variables are of concern is discussed by Bennett (1980), Dalal (1978), Dalal and Fortini (1983), James (1954), Johnson and Weerahandi (1988), Kim (1992), Siotani (1987), Subrahmaniam and Subrahmaniam (1973, 1975), and Yao (1965).

In this review, we have focused on parametric solutions to the Behrens-Fisher problem. Nonparametric or distribution-free solutions have also been proposed for this type of problem (see Lehmann, 1975). In general, these methods attempt to evaluate the strength of the information in the data by replacing observed values with ranks or using permutations of the data.

The Behrens-Fisher problem itself is concerned with only one of the assumptions of the t test (or the analysis of variance), namely, equality of variances. Other assumptions are also important for comparison of group means. Speicifically, assumptions concerned with the model, independence, and normality need to be considered in addition to the equal variance assumption.

References

Aspin, A. A. (1948). An examination and further development of a formula arising in the problem of comparing two mean values. *Biometrika*, *35*, 88–96.

Aspin, A. A. (1949). Tables for use in comparisons whose accuracy involves two variances, separately estimated. *Biometrika*, *36*, 290–296.

Barnard, G. A. (1984). Comparing the means of two independent samples. *Applied Statistics*, *33*, 266–271.

Banerjee, S. K. (1960). Approximate confidence interval for linear functions of means of k populations when the population variances are not equal. *Sankhyā*, *22*, 357–358.

Banerjee, S. K. (1961). On confidence interval for two-means problem based on separate estimates of variances and tabulated values of t-table. *Sankhyā*, *23*, 359–378.

Bartlett, M. S. (1936). The information available in small samples. *Proceedings of the Cambridge Philosophical Society*, *32*, 560–566.

Behrens, W. U. (1929). Ein Beitrag zur Fehlerberechnung bei wenigen Beobachtungen [A contribution to error estimation with few observations]. *Landwirtschaftliche Jahrbücher*, *68*, 807–837.

Bennett, B. M. (1950). Note on a solution of the generalized Behrens-Fisher problem. *Annals of the Institute of Statistical Mathematics*, *2*, 87–90.

Berger, J. O. (1985). *Statistical decision theory and Bayesian analysis* (2nd ed.). New York: Springer-Verlag.

Best, D. J., & Rayner, J. C. W. (1987). Welch's approximate solution for the Behrens-Fisher problem. *Technometrics*, *29*, 205–210.

Box, G. E. P., & Tiao, G. C. (1973). *Bayesian inference in statistical analysis*. Reading, MA: Addison-Wesley.

Broemeling, L. D., & Son, M. S., & Hamdy, H. I. (1990). A Bayesian solution to the Behrens-Fisher problem. In S. Geisser, J. S. Hodges, S. J. Press, & A. Zellner (Eds.), *Bayesian and Likelihood methods in statistics and econometrics: Essay in honor of George A. Barnard* (pp. 229–239). Amsterdam: North-Holland.

Brown, M. B., & Forsythe, A. B. (1974). The small sample behavior of some statistics which test the equality of several means. *Technometrics*, *16*, 129–132.

Clinch, J. C., & Kesselman, H. J. (1982). Parametric alternatives to the analysis of variance. *Journal of Educational Statistics*, 7, 207–214.

Cochran, W. G. (1964). Approximate significance levels of the Behrens-Fisher test. *Biometrics*, *20*, 191–195.

Cochran, W. G., & Cox, G. M. (1950). *Experimental designs*. New York: Wiley.

Dalal, S. R. (1978). Simultaneous confidence procedures for univariate and multivariate Behrens-Fisher type problems. *Biometrika*, *65*, 221–225.

Dalal, S. R., & Fortini, P. (1982). An inequality comparing sums and maxima with application to Behrens-Fisher type problems. *The Annals of Statistics*, *10*, 297–301.

Davenport, J. M., & Webster, J. T. (1975). The Behrens-Fisher problem, and old solution revisited. *Metrika*, *22*, 47–54.

Dayal, H. H., & Dickey, J. M. (1976). Bayes factors for Behrens-Fisher problems. *Sankhyā*, *Series B*, *38*, 315–328.

Finney, D. J. (1952). *Statistical method in biological assay*. New York: Hafner.

Fisher, R. A. (1935). The fiducial argument in statistical inference. *Annals of Eugenics*, *6*, 391–398.

Fisher, R. A. (1939). The comparison of samples with possibly unequal variances. *Annals of Eugenics*, *9*, 174–180.

Fisher, R. A. (1941). The asymptotic approach to Behrens's integral, with further tables for the *d* test of significance. *Annals of Eugenics*, *11*, 141–172.

Fisher, R. A. (1956). On a test of significance in Pearson's *Biometrika tables* (no. 11). *Journal of the Royal Statistical Society, Series B*, *18*, 56–60.

Fisher, R. A., & Healy, M. J. R. (1956). New tables of Behrens' test of significance. *Journal of the Royal Statistical Society, Series B*, *18*, 212–216.

Fisher, R. A., & Yates, F. (1957). *Statistical tables for biological, agricultural and medical research* (4th ed.). Edinburgh, England: Oliver and Boyd.

Fraser, D. A. S. (1978). Fiducial inference. In W. H. Kruskal & J. M. Tanur (Eds.), *International encyclopedia of statistics* (Vol. 1, pp. 349–352). New York: The Free Press.

Galen Research (1992). *Electronic tables* [Computer program]. Salt Lake City, UT: S. E. Warner Software.

Ghosh, B. K. (1975). On the distribution of the difference of two *t*-variables. *Journal of the American Statistical Association*, *70*, 463–467.

Greenwood, J. A., & Hartley, H. O. (1962). *Guide to tables in mathematical statistics*. Princeton, NJ: Princeton University Press.

Gronow, D. G. C. (1951). Test for the significance of the difference between means in two normal populations having unequal variance. *Biometrika*, *38*, 252–256.

Hsu, P. L. (1938). Contribution to the theory of "Student's" *t*-test as applied to the problem of two samples. *Statistical Research Memoirs*, *2*, 1–24.

Isaacs, G. L., Christ, D. E., Novick, M. R., & Jackson, P. H. (1974). *Tables for Bayesian statisticians*. Iowa City: The University of Iowa.

James, G. S. (1951). The comparison of several groups of observations when the ratio of the population variances are unknown. *Biometrika*, *38*, 324–329.

James, G. S. (1954). Tests of linear hypotheses in univariate and multivariate analysis when the ratios of the population variances are unknown. *Biometrika*, *41*, 19–43.

James, G. S. (1959). The Behrens-Fisher distribution and weighted means. *Journal of the Royal Statistical Society, Series B*, *21*, 73–90.

Jeffreys, H. (1940). Note on the Behrens-Fisher formula. *Annals of Eugenics*, *10*, 48–51.

Jeffreys, H. (1961). *Theory of probability* (3rd ed.). London: Oxford University Press.

Johansen, S. (1980). The Welch-James approximation to the distribution of the residual sum of squares in a weighted linear regression. *Biometrika*, *67*, 85–92.

Johnson, N. L., Kotz, S., & Balakrishnan, N. (1995). *Continuous univariate distributions* (Vol. 2, 2nd ed.). New York: Wiley.

Johnson, R. A., & Weerahandi, S. (1988). A Bayesian solution to the multivariate Behrens-Fisher problem. *Journal of the American Statistical Association*, *83*, 145–149.

Kendall, M., & Stuart, A. (1979). *The advanced theory of statistics* (Vol. 2, 4th ed.). New York: Oxford University Press.

Kim, S.-J. (1992). A practical solution to the multivariate Behrens-Fisher problem. *Biometrika*, *79*, 171-176.

Lee, A. F. S., & Gurland, J. (1975). Size and power of tests for equality of means of two normal populations with unequal variances. *Journal of the American Statistical Association*, *70*, 933–941.

Lee, P. M. (1989). *Bayesian statistics: An introduction*. Oxford, England: Oxford University Press.

Lehmann, E. L. (1975). *Nonparametrics: Statistical methods based on ranks*. San Francisco: Holden-Day.

Lehmann, E. L. (1986). *Testing statistical hypotheses* (2nd ed.). New York: Wiley.

Lehmann, E. L. (1993). The Fisher, Neyman-Pearson theories of testing hypotheses: One theory or two? *Journal of the American Statistical Association*, *88*, 1242–1249.

Lindley, D. V. (1965). *Introduction to probability and statistics from Bayesian view point* (Parts 1 & 2). Cambridge, England: Cambridge University Press.

Lindley, D. V., & Scott, W. F. (1984). *New Cambridge elementary statistical tables*. Cambridge, England: Cambridge University Press.

Linnik, J. V. (1968). *Statistical problems with nuisance parameters* (Scripta Technica, Trans.). Providence, RI: American Mathematical Society. (Original work published 1966)

Linssen, H. N. (1991). A table for solving the Behrens-Fisher problem. *Statistics & Probability Letters*, *11*, 359–363.

Marascuilo, L. A., & Serlin, R. C. (1988). *Statistical methods for the social and behavioral sciences*. New York: W. H. Freeman.

Mehta, J. S., & Srinivasan, R. (1970). On the Behrens-Fisher problem. *Biometrika*, *57*, 649–655.

Mickey, M. R., & Brown, M. B. (1966). Bounds on the distribution functions of the Behrens-Fisher statistic. *The Annals of Mathematical Statistics*, *37*, 639–642.

Molenaar, I. W. (1979). Simple approximations to the Behrens-Fisher distribution. *Journal of Statistical Computation and Simulation*, *9*, 283–288.

Nel, D. G., van der Merwe, C. A., & Moser, B. K. (1990). The exact distributions of the univariate and multivariate Behrens-Fisher statistics with a comparison of several solutions in the univariate case. *Communications in Statistics-Theory and Methods*, *19*, 279–298.

Neyman, J., & Pearson, E. S. (1928). On the use and interpretation of certain test criteria for purposes of statistical inference. *Biometrika*, *20A*, 175–240, 263–294.

Neyman, J., & Pearson, E. S. (1933). On the problem of the most efficient test of statistical hypotheses. *Philosophical Transactions of the Royal Society of London, Series A, 231*, 289–337.

Novick, M. R., & Jackson, P. H. (1974). *Statistical methods for educational and psychological research*. New York: McGraw-Hill.

Pagurova, V. I. (1968). On a comparison of means of two normal samples (H. Herman Trans.). *Theory of Probability and Its Applications, 13*, 527–534.

Patil, V. H. (1964). The Behrens-Fisher problem and its Bayesian solution. *Journal of the Indian Statistical Association, 2*, 21–31.

Patil, V. H. (1965). Approximation to the Behrens-Fisher distributions. *Biometrika, 52*, 267–271.

Pearson, E. S., & Hartley, H. O. (Eds.). (1970). *Biometrika tables for statisticians* (Vol. 1). Cambridge, England: Cambridge University Press.

Rahman, M., & Saleh, A. K. M. E. (1974). Explicit form of the distribution of the Behrens-Fisher d-statistic. *Journal of the Royal Statistical Society, Series B, 36*, 54–60; Corrigendum, 466.

Robinson, G. K. (1976). Properties of Student's t and of the Behrens-Fisher solution to the two means problem. *The Annals of Statistics, 4*, 963–971; Correction (1982), *10*, 321.

Robinson, G. K. (1982). Behrens-Fisher problem. In S. Kotz & N. L. Johnson (Eds.), *Encyclopedia of statistical sciences* (Vol. 1, pp. 205–208). New York: Wiley.

Ruben, H. (1960). On the distribution of the weighted difference of two independent student variables. *Journal of the Royal Statistical Society, Series B, 22*, 188–194.

Satterthwaite, F. E. (1941). Synthesis of variance. *Psychometrika, 6*, 309–316.

Satterthwaite, F. E. (1946). An approximate distribution of estimates of variance components. *Biometric Bulletin, 2*, 110–114.

Scheffé, H. (1943). On solutions of the Behrens-Fisher problem based on the t-distribution. *The Annals of Mathematical Statistics*, *14*, 35–44.

Scheffé, H. (1970). Practical solutions of the Behrens-Fisher problem. *Journal of the American Statistical Association*, *65*, 1501–1508.

Siotani, M. (1987). Multivariate Behrens-Fisher problem by heteroscedastic method. In A. K. Gupta (Ed.), *Advances in multivariate statistical analysis* (pp. 327–340). Dordrecht, The Netherlands: Reidel.

Smith, H. F. (1936). The problem of comparing the results of two experiments with unequal errors. *Journal of the Council for Scientific and Industrial Research*, *9*, 211–212.

Sprott, D. A., & Farewell, V. T. (1993). The difference between two normal means. *The American Statistician*, *47*, 126–128.

Subrahmaniam, K., & Subrahmaniam, K. (1973). On the multivariate Behrens-Fisher problem. *Biometrika*, *60*, 107–111.

Subrahmaniam, K., & Subrahmaniam, K. (1975). On the confidence region comparison of some solutions to the multivariate Behrens-Fisher problem. *Communications in Statistics*, *4*, 57–67.

Sukhatme, P. V. (1938). On Fisher and Behrens' test of significance for the difference in means of two normal samples. *Sankhyā*, *4*, 39–48.

Sukhatme, P. V., Thawani, V. D., Pendharkar, V. G., & Natu, N. P. (1951). Revised tables for the d-test of significance. *Journal of the Indian Society of Agricultural Statistics*, *3*, 9–23.

Tamhane, A. C. (1977). Multiple comparisons in model I one-way ANOVA with unequal variances. *Communications in Statistics-Theory and Methods*, *A6*, 15–22.

Trickett, W. H., Welch, B. L., & James, G. S. (1956). Further critical values for the two-means problem. *Biometrika*, *43*, 203–205.

Tsakok, A. D. (1978). A solution to the generalized Behrens-Fisher problem. *Metron*, *36*(3-4), 79–91.

Tsui, K.-H., & Weerahandi, S. (1989). Generalized p-values in significance testing of hypotheses in the presence of nuisance parameters. *Journal of the American Statistical Association*, *84*, 602–607; Correction, *86*, 256.

Wallace, D. L. (1980). The Behrens-Fisher and Fieller-Creasy problems. In S. E. Fienberg & D. V. Hinkley (Eds.), *R. A. Fisher: An appreciation* (pp. 119–147). Berlin: Springer-Verlag.

Wald, A. (1955). Testing the difference between the means of two normal populations with unknown standard deviations. In *Selected papers in statistics and probability by Abraham Wald* (pp. 669–695). New York: McGraw-Hill.

Wang, Y. Y. (1971). Probability of the type I error of the Welch tests for the Behrens-Fisher problem. *Journal of the American Statistical Association*, *66*, 605–608.

Weir, J. B. (1966). Table of 0.1 percentage points of Behrens's d. *Biometrika*, *53*, 267–268.

Welch, B. L. (1938). The significance of the difference between two means when the population variances are unequal. *Biometrika*, *29*, 350–362.

Welch, B. L. (1947). The generalization of 'Student's' problem when several different population variances are involved. *Biometrika*, *34*, 28–35.

Welch, B. L. (1951). On the comparison of several mean values: An alternative approach. *Biometrika*, *38*, 330–336.

Welch, B. L. (1956). Note on some criticisms made by Sir Ronald Fisher. *Journal of the Royal Statistical Society, Series B*, *18*, 297–302.

Yao, Y. (1965). An approximate degrees of freedom solution to the multivariate Behrens-Fisher problem. *Biometrika*, *52*, 139–147.

Table 1

Values of t_1, Values of t_0 Corresponding to $\tau_0 = 1.99704$, Probability of t_2 Exceeding the Values of t_0, and Values of $f(c_k)\Delta t_1$

k	t_1	t_0	$\Pr\{t_2 > t_0\}$	$f(c_k)\Delta t_1$	k	t_1	t_0	$\Pr\{t_2 > t_0\}$	$f(c_k)\Delta t_1$
1	-10.3	-7.4758	0.9999999998074004	0.0000000000000013	76	-2.8	0.0242	0.4903795421123505	0.0004601789451886
2	-10.2	-7.3758	0.9999999997145280	0.0000000000000019	77	-2.7	0.1242	0.4507703483104706	0.0005422214883407
3	-10.1	-7.2758	0.9999999995769073	0.0000000000000028	78	-2.6	0.2242	0.4116650223731995	0.0006302436251987
4	-10.0	-7.1758	0.9999999993730434	0.0000000000000040	79	-2.5	0.3242	0.3734413385391235	0.0007223795934042
5	-9.9	-7.0758	0.9999999990711576	0.0000000000000059	80	-2.4	0.4242	0.3364536166191101	0.0008162114076970
6	-9.8	-6.9758	0.9999999986243348	0.0000000000000087	81	-2.3	0.5242	0.3010220229625702	0.0009088398789208
7	-9.7	-6.8758	0.9999999979633403	0.0000000000000128	82	-2.2	0.6242	0.2674189507961273	0.0009969950216740
8	-9.6	-6.7758	0.9999999969861362	0.0000000000000187	83	-2.1	0.7242	0.2358667999505997	0.0010772182146061
9	-9.5	-6.6758	0.9999999955425156	0.0000000000000275	84	-2.0	0.8242	0.2065342664718628	0.0011460812503546
10	-9.4	-6.5758	0.9999999934115884	0.0000000000000405	85	-1.9	0.9242	0.1795323342084885	0.0012004127786775
11	-9.3	-6.4758	0.9999999902689779	0.0000000000000596	86	-1.8	1.0242	0.1549178212881088	0.0012375481626864
12	-9.2	-6.3758	0.9999999856391194	0.0000000000000879	87	-1.7	1.1242	0.1326952874660492	0.0012555365441612
13	-9.1	-6.2758	0.9999999788257462	0.0000000000001296	88	-1.6	1.2242	0.1128244549036026	0.0012533292346452
14	-9.0	-6.1758	0.9999999688112506	0.0000000000001912	89	-1.5	1.3242	0.0952238813042641	0.0012308582074638
15	-8.9	-6.0758	0.9999999541113240	0.0000000000002823	90	-1.4	1.4242	0.0797796100378036	0.0011890672825676
16	-8.8	-5.9758	0.9999999325651986	0.0000000000004171	91	-1.3	1.5242	0.0663524568080902	0.0011298453774101
17	-8.7	-5.8758	0.9999999010332292	0.0000000000006166	92	-1.2	1.6242	0.0547847151756287	0.0010558691268186
18	-8.6	-5.7758	0.9999998549650903	0.0000000000009119	93	-1.1	1.7242	0.0449065745218661	0.0009703755630239
19	-8.5	-5.6758	0.9999997877814396	0.0000000000013494	94	-1.0	1.8242	0.0365479651420799	0.0008770325899904
20	-8.4	-5.5758	0.9999996899924781	0.0000000000019976	95	-0.9	1.9242	0.0295353267152739	0.0007795009930811
21	-8.3	-5.4758	0.9999995479502213	0.0000000000029582	96	-0.8	2.0242	0.0237019982305681	0.0006813057965225
22	-8.2	-5.3758	0.9999993420794987	0.0000000000043822	97	-0.7	2.1242	0.0188900765293245	0.0005855943629840
23	-8.1	-5.2758	0.9999990443940153	0.0000000000064934	98	-0.6	2.2242	0.0149531986434116	0.0004949884343997
24	-8.0	-5.1758	0.9999986150161370	0.0000000000096239	99	-0.5	2.3242	0.0117580384572025	0.0004114881290667
25	-7.9	-5.0758	0.9999979972949784	0.0000000000142658	100	-0.4	2.4242	0.0091851930415962	0.0003364434241982
26	-7.8	-4.9758	0.9999971110910831	0.0000000000211487	101	-0.3	2.5242	0.0071293306789205	0.0002705789668534
27	-7.7	-4.8758	0.9999958434018565	0.0000000000313536	102	-0.2	2.6242	0.0054988834658188	0.0002140652462193
28	-7.6	-4.7758	0.9999940355777606	0.0000000000464812	103	-0.1	2.7242	0.0042152556226653	0.0001666156522453
29	-7.5	-4.6758	0.9999914658027664	0.0000000000689007	104	0.0	2.8242	0.0032118789926099	0.0001276023097736
30	-7.4	-4.5758	0.9999878253301161	0.0000000001021165	105	0.1	2.9242	0.0024330053800264	0.0000961689668664
31	-7.3	-4.4758	0.9999826863375657	0.0000000001513080	106	0.2	3.0242	0.0018324853230374	0.0000713365584682
32	-7.2	-4.3758	0.9999754588357236	0.0000000002241241	107	0.3	3.1242	0.0013725146028222	0.0000520909465345
33	-7.1	-4.2758	0.9999653340486624	0.0000000003318493	108	0.4	3.2242	0.0010224484513564	0.0000374511517050
34	-7.0	-4.1758	0.9999512076793717	0.0000000004911149	109	0.5	3.3242	0.0007576681098542	0.0000265155990187
35	-6.9	-4.0758	0.9999315816516018	0.0000000007264028	110	0.6	3.4242	0.0005585987612027	0.0000184910220789
36	-6.8	-3.9758	0.9999044354308215	0.0000000010737053	111	0.7	3.5242	0.0004090166533794	0.0000127038947483
37	-6.7	-3.8758	0.9998670599265634	0.0000000015858611	112	0.8	3.6242	0.0002992041410489	0.0000086005202455
38	-6.6	-3.7758	0.9998158458544305	0.0000000023403200	113	0.9	3.7242	0.0002174472281806	0.0000057389014838
39	-6.5	-3.6758	0.9997460179377756	0.0000000034504125	114	1.0	3.8242	0.0001573256707092	0.0000037753056817
40	-6.4	-3.5758	0.9996512994407228	0.0000000050816659	115	1.1	3.9242	0.0001133384924592	0.0000024491047188
41	-6.3	-3.4758	0.9995235043837037	0.0000000074753552	116	1.2	4.0242	0.0000813117020256	0.0000015671253477
42	-6.2	-3.3758	0.9993520337056183	0.0000000109823946	117	1.3	4.1242	0.0000581029843876	0.0000009893738903
43	-6.1	-3.2758	0.9991232702249710	0.0000000161119389	118	1.4	4.2242	0.0000413598664141	0.0000006164440255
44	-6.0	-3.1758	0.9988198746063472	0.0000000236008185	119	1.5	4.3242	0.0000293335634539	0.0000003791638908
45	-5.9	-3.0758	0.9984199467240367	0.0000000345123296	120	1.6	4.4242	0.0000207311379359	0.0000002302952960
46	-5.8	-2.9758	0.9978960766046134	0.0000000503761484	121	1.7	4.5242	0.0000146022126525	0.0000001381632457
47	-5.7	-2.8758	0.9972142999315584	0.0000000733854940	122	1.8	4.6242	0.0000102522197575	0.0000000818990070
48	-5.6	-2.7758	0.9963329074268792	0.0000001066734153	123	1.9	4.7242	0.0000071760467463	0.0000000479814305
49	-5.5	-2.6758	0.9952012631132817	0.0000001546975706	124	2.0	4.8242	0.0000050082370269	0.0000000277912554
50	-5.4	-2.5758	0.9937585140501248	0.0000002237724083	125	2.1	4.9242	0.0000034855966184	0.0000000159189346
51	-5.3	-2.4758	0.9919324216732749	0.0000003227995931	126	2.2	5.0242	0.0000024195011720	0.0000000090204176
52	-5.2	-2.3758	0.9896382051805575	0.0000004642618939	127	2.3	5.1242	0.0000016752887984	0.0000000050579996
53	-5.1	-2.2758	0.9867776543472090	0.0000006655625853	128	2.4	5.2242	0.0000011572558370	0.0000000028074164
54	-5.0	-2.1758	0.9832386802435186	0.0000009508110460	129	2.5	5.3242	0.0000007976275627	0.0000000015429194
55	-4.9	-2.0758	0.9788950209400916	0.0000013531738095	130	2.6	5.4242	0.0000005486063992	0.0000000008398957
56	-4.8	-1.9758	0.9736089577857244	0.0000019179279186	131	2.7	5.5242	0.0000003765879031	0.0000000004529891
57	-4.7	-1.8758	0.9672221439644469	0.0000027063622726	132	2.8	5.6242	0.0000002580291761	0.0000000002421381
58	-4.6	-1.7758	0.9595782657125062	0.0000038006749658	133	2.9	5.7242	0.0000001764914261	0.0000000001283183
59	-4.5	-1.6758	0.9505039080977440	0.0000053099771423	134	3.0	5.8242	0.0000001205258975	0.0000000000674366
60	-4.4	-1.5758	0.9398287981748581	0.0000073775241894	135	3.1	5.9242	0.0000000821845505	0.0000000000351575
61	-4.3	-1.4758	0.9273800328373909	0.0000101890384594	136	3.2	6.0242	0.0000000559630363	0.0000000000181881
62	-4.2	-1.3758	0.9129935503005981	0.0000139820754802	137	3.3	6.1242	0.0000000380591614	0.0000000000093396
63	-4.1	-1.2758	0.8965199738740921	0.0000190559511796	138	3.4	6.2242	0.0000000258531309	0.0000000000047619
64	-4.0	-1.1758	0.8778311237692833	0.0000257815431157	139	3.5	6.3242	0.0000000175431271	0.0000000000024114
65	-3.9	-1.0758	0.8568279147148132	0.0000346099243405	140	3.6	6.4242	0.0000000118927505	0.0000000000012131
66	-3.8	-0.9758	0.8334480077028275	0.0000460783473622	141	3.7	6.5242	0.0000000080552957	0.0000000000006065
67	-3.7	-0.8758	0.8076722323894501	0.0000608116343943	142	3.8	6.6242	0.0000000054518982	0.0000000000003014
68	-3.6	-0.7758	0.7795300632715225	0.0000795166717817	143	3.9	6.7242	0.0000000036873826	0.0000000000001489
69	-3.5	-0.6758	0.7491041123867035	0.0001029675099191	144	4.0	6.8242	0.0000000024924814	0.0000000000000732
70	-3.4	-0.5758	0.7165333032608032	0.0001319786723573	145	4.1	6.9242	0.0000000016839375	0.0000000000000358
71	-3.3	-0.4758	0.6820122897624969	0.0001673643362528	146	4.2	7.0242	0.0000000011371967	0.0000000000000174
72	-3.2	-0.3758	0.6457884609699249	0.0002098821188380	147	4.3	7.1242	0.0000000007677066	0.0000000000000084
73	-3.1	-0.2758	0.6081587076187134	0.0002601624400930	148	4.4	7.2242	0.0000000005181303	0.0000000000000041
74	-3.0	-0.1758	0.5694607794284821	0.0003186244129421	149	4.5	7.3242	0.0000000003496223	0.0000000000000020
75	-2.9	-0.0758	0.5300712287425995	0.0003853889176623					

$\Pr\{\tau > \tau_0\} = \sum f(c_k)\Delta t_1 = 0.025$

BEST COPY AVAILABLE

Table 2

Percentage Points of the Behrens-Fisher Distribution for $\nu_1, \nu_2 = 1(1)8, 10, 12, 24, \infty$, $\theta = 0°(15°)90°$, $\alpha = .10, .05, .02, .01$

ν_1, ν_2	α	0°	15°	30°	45°	60°	75°	90°
1,1	.10	6.31375	7.73273	8.62474	8.92899	8.62474	7.73273	6.31375
	.05	12.70620	15.86186	17.35700	17.96929	17.35700	15.86186	12.70620
	.02	31.82052	38.97201	43.46763	45.00101	43.46763	38.97201	31.82052
	.01	63.65674	77.96326	86.93672	90.02423	86.93672	77.96326	63.65674
1,2	.10	2.91999	3.92422	4.81108	5.51485	5.99718	6.25177	6.31375
	.05	4.30265	6.31134	8.32614	10.12725	11.53258	12.41375	12.70620
	.02	6.96455	12.20822	18.13899	23.66761	28.06059	30.86235	31.82052
	.01	9.92482	21.05806	34.14512	46.16851	55.61373	61.60681	63.65674
1,3	.10	2.35336	3.23550	4.12401	4.98809	5.65934	6.13872	6.31375
	.05	3.18245	4.95977	7.12269	9.30312	11.11208	12.29429	12.70620
	.02	4.54070	9.52070	16.30672	22.64110	27.60237	30.74479	31.82052
	.01	5.84091	17.28287	32.03669	45.08352	55.18098	61.49196	63.65674
1,4	.10	2.13185	2.95933	3.86159	4.77767	5.57804	6.12159	6.31375
	.05	2.77645	4.46447	6.77038	9.13618	11.05568	12.28450	12.70620
	.02	3.74695	8.80613	16.04749	22.55223	27.57655	30.74067	31.82052
	.01	4.60409	16.70163	31.88659	45.03612	55.13767	61.48988	63.65674
1,5	.10	2.01505	2.81232	3.72983	4.70125	5.55092	6.11690	6.31375
	.05	2.57058	4.21758	6.63646	9.09012	11.04305	12.28235	12.70620
	.02	3.36493	8.56288	15.99780	22.53854	27.57259	30.73989	31.82052
	.01	4.03214	16.59318	31.86885	45.03075	55.13592	61.48950	63.65674
1,6	.10	1.94318	2.72152	3.65305	4.66300	5.53921	6.11488	6.31375
	.05	2.44691	4.07337	6.57654	9.07314	11.03833	12.28141	12.70620
	.02	3.14267	8.46783	15.98355	22.53408	27.57100	30.73953	31.82052
	.01	3.70742	16.56656	31.86396	45.02880	55.13516	61.48933	63.65674
1,7	.10	1.89458	2.66006	3.60383	4.64137	5.53302	6.11373	6.31375
	.05	2.36462	3.98044	6.54601	9.06502	11.03582	12.28086	12.70620
	.02	2.99795	8.42578	15.97736	22.53172	27.57008	30.73931	31.82052
	.01	3.49948	16.55705	31.86146	45.02768	55.13470	61.48922	63.65674
1,8	.10	1.85955	2.61576	3.57007	4.62794	5.52924	6.11299	6.31375
	.05	2.30600	3.91637	6.52864	9.06031	11.03422	12.28049	12.70620
	.02	2.89646	8.40475	15.97378	22.53020	27.56947	30.73916	31.82052
	.01	3.35538	16.55224	31.85986	45.02693	55.13440	61.48914	63.65674
1,10	.10	1.81246	2.55628	3.52744	4.61261	5.52484	6.11207	6.31375
	.05	2.22814	3.83503	6.51053	9.05496	11.03227	12.28004	12.70620
	.02	2.76377	8.38556	15.96958	22.52832	27.56871	30.73898	31.82052
	.01	3.16927	16.54700	31.85787	45.02600	55.13402	61.48904	63.65674
1,12	.10	1.78229	2.51825	3.50208	4.60428	5.52234	6.11152	6.31375
	.05	2.17881	3.78641	6.50145	9.05193	11.03111	12.27977	12.70620
	.02	2.68100	8.37694	15.96714	22.52720	27.56826	30.73887	31.82052
	.01	3.05454	16.54403	31.85669	45.02545	55.13380	61.48900	63.65674
1,24	.10	1.71088	2.42872	3.44935	4.58825	5.51708	6.11033	6.31375
	.05	2.06390	3.68543	6.48488	9.04552	11.02860	12.27917	12.70620
	.02	2.49216	8.36187	15.96185	22.52476	27.56727	30.73864	31.82052
	.01	2.79694	16.53765	31.85410	45.02424	55.13331	61.48889	63.65674
1,∞	.10	1.64481	2.34652	3.40944	4.57412	5.50986	6.10602	6.31375
	.05	1.95991	3.61340	6.46939	9.03548	11.02059	12.27205	12.70620
	.02	2.32631	8.34623	15.94896	22.51063	27.55161	30.72189	31.82052
	.01	2.57581	16.52357	31.83484	44.99901	55.10323	61.45570	63.65674
2,1	.10	6.31375	6.25177	5.99718	5.51485	4.81108	3.92422	2.91999
	.05	12.70620	12.41375	11.53258	10.12725	8.32614	6.31134	4.30265
	.02	31.82052	30.86235	28.06059	23.66761	18.13899	12.20822	6.96455
	.01	63.65674	61.60681	55.61373	46.16851	34.14512	21.05806	9.92482
2,2	.10	2.91999	3.03383	3.17315	3.22807	3.17315	3.03383	2.91999
	.05	4.30265	4.41381	4.56288	4.62396	4.56288	4.41381	4.30265
	.02	6.96455	7.06123	7.20468	7.26645	7.20468	7.06123	6.96455
	.01	9.92482	10.00723	10.13676	10.19433	10.13676	10.00723	9.92482
2,3	.10	2.35336	2.49530	2.68859	2.82874	2.89730	2.91527	2.91999
	.05	3.18245	3.36047	3.64523	3.90252	4.10025	4.23991	4.30265
	.02	4.54070	4.79100	5.28466	5.84165	6.37504	6.79275	6.96455
	.01	5.84091	6.18733	6.96557	7.93732	8.90495	9.64006	9.92482
2,4	.10	2.13185	2.28310	2.49609	2.67337	2.79942	2.88351	2.91999
	.05	2.77645	2.97833	3.31242	3.65282	3.96364	4.20522	4.30265
	.02	3.74695	4.05358	4.67802	5.44249	6.20064	6.75946	6.96455
	.01	4.60409	5.04853	6.08192	7.42809	8.71689	9.60944	9.92482
2,5	.10	2.01505	2.17060	2.39404	2.59327	2.75331	2.87135	2.91999
	.05	2.57058	2.78415	3.14475	3.53453	3.90859	4.19447	4.30265
	.02	3.36493	3.70001	4.39889	5.28626	6.14924	6.75199	6.96455
	.01	4.03214	4.52754	5.71647	7.26976	8.67575	9.60415	9.92482
2,6	.10	1.94318	2.10111	2.33119	2.54526	2.72770	2.86541	2.91999
	.05	2.44691	2.66731	3.04504	3.46839	3.88168	4.18979	4.30265
	.02	3.14267	3.49489	4.24462	5.21308	6.12942	6.74909	6.96455
	.01	3.70742	4.23461	5.53489	7.21002	8.66265	9.60219	9.92482
2,7	.10	1.89458	2.05401	2.28874	2.51361	2.71182	2.86199	2.91999
	.05	2.36462	2.58948	2.97944	3.42726	3.86654	4.18721	4.30265
	.02	2.99795	3.36170	4.14936	5.17411	6.11981	6.74752	6.96455
	.01	3.49948	4.04879	5.43399	7.18321	8.65671	9.60111	9.92482
2,8	.10	1.85955	2.01999	2.25821	2.49131	2.70118	2.86078	2.91999
	.05	2.30600	2.53399	2.93322	3.39970	3.85710	4.18588	4.30265
	.02	2.89646	3.26852	4.08594	5.15113	6.11425	6.74650	6.96455
	.01	3.35538	3.92122	5.37322	7.16912	8.65326	9.60041	9.92482

Table 2-continued

Percentage Points of the Behrens-Fisher Distribution for $\nu_1, \nu_2 = 1(1)8, 10, 12, 24, \infty$, $\theta = 0°(15°)90°$, $\alpha = .10, .05, .02, .01$

ν_1, ν_2	α	0°	15°	30°	45°	60°	75°	90°
2,10	.10	1.81246	1.97418	2.21731	2.46219	2.68798	2.85710	2.91999
	.05	2.22814	2.46025	2.87275	3.36571	3.84615	4.18361	4.30265
	.02	2.76377	3.14714	4.00853	5.12639	6.10803	6.74526	6.96455
	.01	3.16927	3.75851	5.30783	7.15506	8.64929	9.59954	9.92482
2,12	.10	1.78229	1.94478	2.19122	2.44413	2.68020	2.85553	2.91999
	.05	2.17881	2.41353	2.83513	3.34591	3.84005	4.18245	4.30265
	.02	2.68100	3.07175	3.96417	5.11370	6.10457	6.74453	6.96455
	.01	3.08454	3.65982	5.27546	7.14800	8.64699	9.59902	9.92482
2,24	.10	1.71088	1.87506	2.13004	2.40367	2.66385	2.85219	2.91999
	.05	2.06390	2.30468	2.75043	3.30555	3.82782	4.17997	4.30265
	.02	2.49216	2.90129	3.87742	5.09067	6.09740	6.74293	6.96455
	.01	2.79694	3.44555	5.22337	7.13443	8.64210	9.59789	9.92482
2,∞	.10	1.64481	1.81026	2.07429	2.36919	2.65028	2.84858	2.91999
	.05	1.95991	2.20631	2.67858	3.27519	3.81739	4.17674	4.30265
	.02	2.32631	2.75461	3.81967	5.07317	6.09000	6.73976	6.96455
	.01	2.57581	3.27599	5.19258	7.12209	8.63577	9.59433	9.92482
3,1	.10	6.31375	6.13872	5.65934	4.95809	4.12401	3.23550	2.35336
	.05	12.70620	12.29429	11.11208	9.30312	7.12269	4.95977	3.18245
	.02	31.82052	30.74479	27.60237	22.64110	16.30672	9.52070	4.54070
	.01	63.65674	61.49196	55.15098	45.08352	32.03669	17.28287	5.84091
3,2	.10	2.91999	2.91527	2.89730	2.82874	2.68859	2.49530	2.35336
	.05	4.30265	4.23991	4.10025	3.90252	3.64523	3.36047	3.18245
	.02	6.96455	6.79275	6.37504	5.84165	5.28466	4.79100	4.54070
	.01	9.92482	9.64006	8.90495	7.93732	6.96557	6.18733	5.84091
3,3	.10	2.35336	2.38618	2.44512	2.47143	2.44512	2.38618	2.35336
	.05	3.18245	3.19135	3.22536	3.24395	3.22536	3.19135	3.18245
	.02	4.54070	4.50128	4.46705	4.45965	4.46705	4.50128	4.54070
	.01	5.84091	5.75394	5.63969	5.59790	5.63969	5.75394	5.84091
3,4	.10	2.13185	2.17969	2.26705	2.33115	2.35447	2.38318	2.35336
	.05	2.77645	2.81572	2.91339	3.01157	3.08779	3.14922	3.18245
	.02	3.74695	3.76498	3.87294	4.04546	4.25029	4.44873	4.54070
	.01	4.60409	4.60093	4.72022	4.98621	5.34942	5.69429	5.84091
3,5	.10	2.01505	2.07070	2.17274	2.25780	2.30968	2.33931	2.35336
	.05	2.57058	2.62621	2.75634	2.89717	3.02568	3.13386	3.18245
	.02	3.36493	3.41494	3.59516	3.86108	4.16752	4.43370	4.54070
	.01	4.03214	4.07640	4.31559	4.73914	5.25584	5.68061	5.84091
3,6	.10	1.94318	2.00356	2.11459	2.21322	2.28375	2.33211	2.35336
	.05	2.44691	2.51265	2.66282	2.83051	2.99219	3.12659	3.18245
	.02	3.14267	3.21288	3.43699	3.76168	4.12866	4.42751	4.54070
	.01	3.70742	3.78213	4.09477	4.61677	5.21781	5.67557	5.84091
3,7	.10	1.89458	1.95810	2.07525	2.18345	2.26712	2.32781	2.35336
	.05	2.36462	2.43719	2.60042	2.78743	2.97188	3.12248	3.18245
	.02	2.99795	3.08203	3.33587	3.70143	4.10751	4.42422	4.54070
	.01	3.49948	3.59533	3.95817	4.54802	5.19922	5.67297	5.84091
3,8	.10	1.85955	1.92531	2.04690	2.16226	2.25566	2.32497	2.35336
	.05	2.30600	2.38349	2.55640	2.75753	2.95848	3.11985	3.18245
	.02	2.89646	2.99063	3.26606	3.66182	4.09465	4.42217	4.54070
	.01	3.35538	3.46675	3.86643	4.50575	5.18866	5.67135	5.84091
3,10	.10	1.81246	1.88118	2.00884	2.13421	2.24102	2.32149	2.35336
	.05	2.22814	2.31222	2.49828	2.71908	2.94213	3.11668	3.18245
	.02	2.76377	2.87164	3.17659	3.61400	4.08010	4.41971	4.54070
	.01	3.16927	3.30189	3.75252	4.45851	5.17731	5.66940	5.84091
3,12	.10	1.78229	1.85288	1.98448	2.11654	2.23214	2.31943	2.35336
	.05	2.17881	2.26711	2.46174	2.69562	2.93265	3.11484	3.18245
	.02	2.68100	2.79772	3.12205	3.58675	4.07219	4.41828	4.54070
	.01	3.08454	3.20103	3.68539	4.43372	5.17128	5.66825	5.84091
3,24	.10	1.71088	1.78580	1.92708	2.07602	2.21275	2.31502	2.35336
	.05	2.06390	2.16214	2.37778	2.64426	2.91307	3.11092	3.18245
	.02	2.49216	2.63011	3.00270	3.53290	4.05667	4.41523	4.54070
	.01	2.79694	2.97720	3.54801	4.38938	5.15940	5.66579	5.84091
3,∞	.10	1.64481	1.72353	1.87431	2.04023	2.19640	2.31086	2.35336
	.05	1.95991	2.06736	2.30375	2.60223	2.89742	3.10707	3.18245
	.02	2.32631	2.48453	2.90609	3.49449	4.04423	4.41186	4.54070
	.01	2.57581	2.78916	3.44907	4.35989	5.14929	5.66269	5.84091
4,1	.10	6.31375	6.12159	5.57804	4.77767	3.86159	2.95933	2.13185
	.05	12.70620	12.28450	11.05868	9.13618	6.77038	4.46447	2.77645
	.02	31.82052	30.74067	27.57655	22.55223	16.04749	8.80613	3.74695
	.01	63.65674	61.48988	55.13767	45.03613	31.88659	16.70163	4.60409
4,2	.10	2.91999	2.88381	2.79942	2.67337	2.49609	2.28310	2.13185
	.05	4.30265	4.20522	3.96364	3.65262	3.31242	2.97833	2.77645
	.02	6.96455	6.75946	6.20064	5.44249	4.67802	4.05358	3.74695
	.01	9.92482	9.60944	8.71689	7.42809	6.08193	5.04853	4.60409
4,3	.10	2.35336	2.35318	2.35447	2.33115	2.26705	2.17969	2.13185
	.05	3.18245	3.14922	3.08779	3.01157	2.91339	2.81572	2.77645
	.02	4.54070	4.44873	4.25029	4.04546	3.87294	3.76498	3.74695
	.01	5.84091	5.69429	5.34942	4.98621	4.72022	4.60093	4.60409
4,4	.10	2.13185	2.14694	2.18050	2.19689	2.18050	2.14694	2.13185
	.05	2.77645	2.77165	2.77927	2.78639	2.77927	2.77165	2.77645
	.02	3.74695	3.70422	3.64912	3.63095	3.64912	3.70422	3.74695
	.01	4.60409	4.52844	4.40009	4.34982	4.40009	4.52844	4.60409

BEST COPY AVAILABLE

Table 2-continued

Percentage Points of the Behrens-Fisher Distribution for $\nu_1, \nu_2 = 1(1)8, 10, 12, 24, \infty$, $\theta = 0°(15°)90°$, $\alpha = .10, .05, .02, .01$

ν_1, ν_2	α	0°	15°	30°	45°	60°	75°	90°
4,5	.10	2.01505	2.03632	2.08856	2.12645	2.13706	2.13265	2.13185
	.05	2.57058	2.58157	2.62466	2.67517	2.71670	2.75444	2.77645
	.02	3.36493	3.35042	3.36966	3.44342	3.55690	3.68460	3.74695
	.01	4.03214	3.99258	3.98331	4.08367	4.28284	4.50502	4.60409
4,6	.10	1.94318	1.97147	2.03190	2.08347	2.11158	2.12505	2.13185
	.05	2.44691	2.46786	2.53244	2.60971	2.68192	2.74594	2.77645
	.02	3.14267	3.14643	3.21067	3.33978	3.51038	3.67602	3.74695
	.01	3.70742	3.69356	3.75511	3.94473	4.22921	4.49701	4.60409
4,7	.10	1.89458	1.92626	1.99358	2.05469	2.09506	2.12042	2.13185
	.05	2.36462	2.39239	2.47141	2.56704	2.66028	2.74101	2.77645
	.02	2.99795	3.01446	3.10884	3.27529	3.48358	3.67137	3.74695
	.01	3.49948	3.50381	3.61293	3.86217	4.20064	4.49289	4.60409
4,8	.10	1.85955	1.89365	1.96596	2.03412	2.08336	2.11734	2.13185
	.05	2.30600	2.33672	2.42813	2.53719	2.64571	2.73783	2.77645
	.02	2.89646	2.92235	3.03835	3.23183	3.46657	3.66846	3.74695
	.01	3.35538	3.37325	3.51656	3.80864	4.18357	4.49036	4.60409
4,10	.10	1.81246	1.84981	1.92885	2.00680	2.06873	2.11352	2.13185
	.05	2.22814	2.26755	2.37094	2.49844	2.62755	2.73396	2.77645
	.02	2.76377	2.80254	2.94756	3.17771	3.44664	3.66502	3.74695
	.01	3.16927	3.20591	3.39522	3.74491	4.16466	4.48737	4.60409
4,12	.10	1.78229	1.82170	1.90509	1.98952	2.05963	2.11124	2.13185
	.05	2.17881	2.22254	2.33493	2.47452	2.61679	2.73170	2.77645
	.02	2.68100	2.72617	2.89185	3.14576	3.43550	3.66302	3.74695
	.01	3.05454	3.10356	3.32252	3.70911	4.15458	4.48564	4.60409
4,24	.10	1.71088	1.75513	1.84908	1.94961	2.03948	2.10634	2.13185
	.05	2.06390	2.11789	2.26191	2.42117	2.59402	2.72690	2.77645
	.02	2.49216	2.55969	2.76817	3.07917	3.41338	3.65881	3.74695
	.01	2.79694	2.87643	3.16698	3.63978	4.13524	4.48197	4.60409
4,∞	.10	1.64481	1.69336	1.79741	1.91399	2.02226	2.10187	2.13185
	.05	1.95991	2.02350	2.17823	2.37625	2.57562	2.72250	2.77645
	.02	2.32631	2.41347	2.66495	3.02862	3.39639	3.65480	3.74695
	.01	2.57581	2.68530	3.04526	3.59200	4.12041	4.47831	4.60409
5,1	.10	6.31375	6.11690	5.55092	4.70125	3.72983	2.81232	2.01505
	.05	12.70620	12.28235	11.04305	9.09012	6.63646	4.21758	2.57058
	.02	31.82052	30.73989	27.57259	22.53854	15.99780	8.86288	3.36493
	.01	63.65674	61.48950	55.13592	45.03075	31.86885	16.59318	4.03214
5,2	.10	2.91999	2.87135	2.75331	2.59327	2.39404	2.17060	2.01505
	.05	4.30265	4.19447	3.90859	3.53483	3.14475	2.78415	2.57058
	.02	6.96455	6.78199	6.14924	5.28626	4.39889	3.70001	3.36493
	.01	9.92482	9.60415	8.67575	7.26976	5.71647	4.52754	4.03214
5,3	.10	2.35336	2.33931	2.30968	2.25780	2.17274	2.07070	2.01505
	.05	3.18245	3.13386	3.02568	2.89717	2.75634	2.62621	2.57058
	.02	4.54070	4.43370	4.16752	3.86108	3.59516	3.41494	3.36493
	.01	5.84091	5.68061	5.25584	4.73914	4.31559	4.07640	4.03214
5,4	.10	2.13185	2.13265	2.13706	2.12645	2.08856	2.03632	2.01505
	.05	2.77645	2.75444	2.71670	2.67517	2.62466	2.58157	2.57058
	.02	3.74695	3.68460	3.55690	3.44342	3.36966	3.35042	3.36493
	.01	4.60409	4.50502	4.28284	4.08367	3.98331	3.99258	4.03214
5,5	.10	2.01505	2.02391	2.04600	2.05747	2.04600	2.02391	2.01505
	.05	2.57058	2.56353	2.56237	2.56518	2.56237	2.56353	2.57058
	.02	3.36493	3.32820	3.27402	3.25431	3.27402	3.32820	3.36493
	.01	4.03214	3.96771	3.85575	3.80866	3.85575	3.96771	4.03214
5,6	.10	1.94318	1.95704	1.98993	2.01532	2.02087	2.01613	2.01505
	.05	2.44691	2.44936	2.47046	2.50038	2.52729	2.55439	2.57058
	.02	3.14267	3.12263	3.11338	3.14914	3.22429	3.31809	3.36493
	.01	3.70742	3.66576	3.62209	3.66328	3.79435	3.95745	4.03214
5,7	.10	1.89458	1.91182	1.95200	1.98705	2.00449	2.01136	2.01505
	.05	2.36462	2.37363	2.40968	2.45802	2.50521	2.54902	2.57058
	.02	2.99795	2.98964	3.01057	3.08317	3.19488	3.31251	3.36493
	.01	3.49948	3.47400	3.47645	3.57550	3.76019	3.93209	4.03214
5,8	.10	1.85955	1.87923	1.92467	1.96682	1.99304	2.00817	2.01505
	.05	2.30600	2.31979	2.36659	2.42830	2.49020	2.54551	2.57058
	.02	2.89646	2.89683	2.93940	3.03836	3.17584	3.30900	3.36493
	.01	3.35538	3.34202	3.37763	3.51764	3.73911	3.94880	4.03214
5,10	.10	1.81246	1.83541	1.88796	1.93990	1.97820	2.00417	2.01505
	.05	2.22814	2.24642	2.30964	2.38957	2.47131	2.54123	2.57058
	.02	2.76377	2.77611	2.84767	2.98197	3.15304	3.30482	3.36493
	.01	3.16927	3.17280	3.25289	3.44726	3.71508	3.94495	4.03214
5,12	.10	1.78229	1.80732	1.86445	1.92285	1.96905	2.00179	2.01505
	.05	2.17881	2.20329	2.27378	2.36556	2.46000	2.53871	2.57058
	.02	2.68100	2.70120	2.79131	2.94825	3.14006	3.30241	3.36493
	.01	3.05454	3.06929	3.17787	3.40670	3.70200	3.94273	4.03214
5,24	.10	1.71088	1.74082	1.80897	1.88333	1.94863	1.99662	2.01505
	.05	2.06390	2.09843	2.19104	2.31161	2.43573	2.53336	2.57058
	.02	2.49216	2.53156	2.66575	2.87652	3.11382	3.29734	3.36493
	.01	2.79694	2.83954	3.01887	3.32500	3.67670	3.93808	4.03214
5,∞	.10	1.64481	1.67915	1.75782	1.84790	1.93106	1.99197	2.01505
	.05	1.95991	2.00392	2.11746	2.26563	2.41590	2.52859	2.57058
	.02	2.32631	2.38438	2.56009	2.82034	3.09366	3.29278	3.36493
	.01	2.57581	2.64614	2.88632	3.26601	3.65784	3.93381	4.03214

Table 2-continued

Percentage Points of the Behrens-Fisher Distribution for $\nu_1, \nu_2 = 1(1)8, 10, 12, 24, \infty$, $\theta = 0°(15°)90°$, $\alpha = .10, .05, .02, .01$

ν_1, ν_2	α	0°	15°	30°	45°	60°	75°	90°
6,1	.10	6.31375	6.11488	5.53921	4.66300	3.65305	2.72152	1.94318
	.05	12.70620	12.28141	11.03833	9.07314	6.57654	4.07337	2.44691
	.02	31.82052	30.73953	27.57100	22.53408	15.98355	8.46783	3.14267
	.01	63.65674	61.48933	55.13516	45.02880	31.86396	16.56656	3.70742
6,2	.10	2.91999	2.86541	2.72770	2.54526	2.33119	2.10111	1.94318
	.05	4.30265	4.18979	3.88168	3.46839	3.04504	2.66731	2.44691
	.02	6.96455	6.74909	6.12942	5.21308	4.24462	3.49489	3.14267
	.01	9.92482	9.60219	8.66265	7.21002	5.53489	4.23461	3.70742
6,3	.10	2.35336	2.33211	2.26375	2.21322	2.11459	2.00356	1.94318
	.05	3.18245	3.12659	2.99219	2.83051	2.66252	2.51265	2.44691
	.02	4.54070	4.42751	4.12866	3.76168	3.43699	3.21288	3.14267
	.01	5.84091	5.67557	5.21781	4.61677	4.09477	3.78213	3.70742
6,4	.10	2.13185	2.12505	2.11158	2.08347	2.03190	1.97147	1.94318
	.05	2.77645	2.74594	2.68192	2.60971	2.53244	2.46786	2.44691
	.02	3.74695	3.67602	3.51038	3.33978	3.21067	3.14643	3.14267
	.01	4.60409	4.49701	4.22921	3.94473	3.75511	3.69356	3.70742
6,5	.10	2.01505	2.01613	2.02087	2.01332	1.98993	1.95704	1.94318
	.05	2.57058	2.55439	2.52729	2.50038	2.47046	2.44936	2.44691
	.02	3.36493	3.31809	3.22429	3.14914	3.11338	3.12263	3.14267
	.01	4.03214	3.95745	3.79435	3.66328	3.62209	3.66576	3.70742
6,6	.10	1.94318	1.94918	1.96505	1.97365	1.96505	1.94918	1.94318
	.05	2.44691	2.43986	2.43531	2.43593	2.43531	2.43986	2.44691
	.02	3.14267	3.11152	3.06203	3.04294	3.06203	3.11152	3.14267
	.01	3.70742	3.65390	3.55642	3.51401	3.55642	3.65390	3.70742
6,7	.10	1.89458	1.90392	1.92731	1.94567	1.94878	1.94433	1.94318
	.05	2.36462	2.36389	2.37451	2.39373	2.41304	2.43422	2.44691
	.02	2.99795	2.97785	2.95825	2.97613	3.03124	3.10530	3.14267
	.01	3.49948	3.46097	3.40815	3.42334	3.51905	3.64760	3.70742
6,8	.10	1.85955	1.87130	1.90011	1.92565	1.93739	1.94107	1.94318
	.05	2.30600	2.30990	2.33142	2.36409	2.39783	2.43051	2.44691
	.02	2.89646	2.88455	2.88643	2.93059	3.01107	3.10137	3.14267
	.01	3.35538	3.32812	3.30751	3.36320	3.49556	3.64372	3.70742
6,10	.10	1.81246	1.82745	1.86358	1.89897	1.92257	1.93698	1.94318
	.05	2.22814	2.23833	2.27449	2.32539	2.37857	2.42597	2.44691
	.02	2.76377	2.76320	2.79386	2.87300	2.98665	3.09667	3.14267
	.01	3.16927	3.15772	3.16042	3.28937	3.46829	3.63918	3.70742
6,12	.10	1.78229	1.79935	1.84019	1.88205	1.91341	1.93453	1.94318
	.05	2.17881	2.19308	2.23864	2.30134	2.36697	2.42329	2.44691
	.02	2.68100	2.68789	2.73697	2.83839	2.97257	3.09396	3.14267
	.01	3.05454	3.05345	3.10388	3.24635	3.45319	3.63658	3.70742
6,24	.10	1.71088	1.73284	1.78498	1.84278	1.89288	1.92921	1.94318
	.05	2.06390	2.08798	2.15592	2.24711	2.34187	2.41758	2.44691
	.02	2.49216	2.51733	2.61009	2.76400	2.94374	3.08826	3.14267
	.01	2.79694	2.82192	2.93811	3.15805	3.42365	3.63114	3.70742
6,∞	.10	1.64481	1.67117	1.73406	1.80747	1.87513	1.92444	1.94318
	.05	1.95991	1.99326	2.08229	2.20060	2.32122	2.41255	2.44691
	.02	2.32631	2.36939	2.50296	2.70481	2.92147	3.08327	3.14267
	.01	2.57581	2.62693	2.80447	3.09253	3.40171	3.62634	3.70742
7,1	.10	6.31375	6.11373	5.53302	4.64137	3.60383	2.66006	1.89458
	.05	12.70620	12.28086	11.03582	9.06502	6.54601	3.98044	2.36462
	.02	31.82052	30.73931	27.57008	22.53172	15.97736	8.42578	2.99795
	.01	63.65674	61.48922	55.13470	45.02768	31.86146	16.55705	3.49948
7,2	.10	2.91999	2.86199	2.71182	2.51361	2.28874	2.05401	1.89458
	.05	4.30265	4.18721	3.86654	3.42726	2.97944	2.58948	2.36462
	.02	6.96455	6.74752	6.11981	5.17411	4.14936	3.36170	2.99795
	.01	9.92482	9.60111	8.65671	7.18321	5.43399	4.04879	3.49948
7,3	.10	2.35336	2.32781	2.26712	2.18345	2.07525	1.95810	1.89458
	.05	3.18245	3.12248	2.97188	2.78743	2.60042	2.43719	2.36462
	.02	4.54070	4.42422	4.10751	3.70143	3.33587	3.08203	2.99795
	.01	5.84091	5.67297	5.19922	4.54802	3.95817	3.59533	3.49948
7,4	.10	2.13185	2.12042	2.09506	2.05469	1.99358	1.92626	1.89458
	.05	2.77645	2.74101	2.66028	2.56704	2.47141	2.39239	2.36462
	.02	3.74695	3.67137	3.48358	3.27529	3.10884	3.01446	2.99795
	.01	4.60409	4.49289	4.20064	3.86217	3.61293	3.50381	3.49948
7,5	.10	2.01505	2.01136	2.00449	1.98705	1.95200	1.91182	1.89458
	.05	2.57058	2.54902	2.50521	2.45802	2.40968	2.37363	2.36462
	.02	3.36493	3.31251	3.19488	3.08317	3.01057	2.98964	2.99795
	.01	4.03214	3.95209	3.76019	3.57550	3.47645	3.47400	3.49948
7,6	.10	1.94318	1.94433	1.94878	1.94567	1.92731	1.90392	1.89458
	.05	2.44691	2.43422	2.41304	2.39373	2.37451	2.36389	2.36462
	.02	3.14267	3.10530	3.03124	2.97613	2.95825	2.97785	2.99795
	.01	3.70742	3.64760	3.51905	3.42334	3.40815	3.46097	3.49948
7,7	.10	1.89458	1.89902	1.91113	1.91788	1.91113	1.89902	1.89458
	.05	2.36462	2.35807	2.35215	2.38161	2.35215	2.35807	2.36462
	.02	2.99795	2.97119	2.92662	2.90869	2.92662	2.97119	2.99795
	.01	3.49948	3.45397	3.36875	3.33071	3.36875	3.45397	3.49948
7,8	.10	1.88955	1.88637	1.88400	1.89798	1.89977	1.89572	1.89458
	.05	2.30600	2.30395	2.30900	2.32201	2.33683	2.35424	2.36462
	.02	2.89646	2.87756	2.85424	2.86265	2.90576	2.96695	2.99795
	.01	3.35538	3.32088	3.26674	3.26908	3.34371	3.44964	3.49948

BEST COPY AVAILABLE

Table 2-continued

Percentage Points of the Behrens-Fisher Distribution for $\nu_1, \nu_2 = 1(1)8, 10, 12, 24, \infty$, $\theta = 0°(15°)90°$, $\alpha = .10, .05, .02, .01$

ν_1, ν_2	α	0°	15°	30°	45°	60°	75°	90°
7,10	.10	1.81246	1.82249	1.84786	1.87145	1.88498	1.89157	1.89458
	.05	2.22814	2.23223	2.25202	2.28331	2.31735	2.34952	2.36462
	.02	2.76377	2.75576	2.76096	2.80430	2.88031	2.96187	2.99795
	.01	3.16927	3.14942	3.13786	3.19308	3.31429	3.44456	3.49948
7,12	.10	1.78229	1.79437	1.82423	1.86462	1.87582	1.88908	1.89458
	.05	2.17881	2.18689	2.21613	2.25924	2.30557	2.34673	2.36462
	.02	2.68100	2.68017	2.70363	2.76912	2.86553	2.95893	2.99795
	.01	3.05454	3.04465	3.06022	3.14854	3.29783	3.44165	3.49948
7,24	.10	1.71088	1.72782	1.76915	1.81550	1.85823	1.88365	1.89458
	.05	2.06390	2.08157	2.13333	2.20483	2.27997	2.34076	2.36462
	.02	2.49216	2.50897	2.57571	2.69309	2.83499	2.95275	2.99795
	.01	2.79694	2.81193	2.89181	3.03619	3.26527	3.43559	3.49948
7,∞	.10	1.64481	1.66612	1.71835	1.78028	1.83737	1.87881	1.89458
	.05	1.95991	1.98668	2.05960	2.15799	2.25878	2.33554	2.36462
	.02	2.32631	2.36045	2.46755	2.63204	2.81124	2.94742	2.99795
	.01	2.57581	2.61584	2.75552	2.98686	3.24102	3.43036	3.49948
8,1	.10	6.31375	6.11299	5.52924	4.62794	3.87007	2.61576	1.85955
	.05	12.70620	12.28049	11.03422	9.06031	6.82864	3.91637	2.30600
	.02	31.82052	30.73916	27.56947	22.53020	15.97378	8.40475	2.89646
	.01	63.65674	61.48914	55.13440	45.02693	31.85986	16.55224	3.35538
8,2	.10	2.91999	2.85978	2.70118	2.49131	2.25821	2.01999	1.85955
	.05	4.30265	4.18558	3.85710	3.39970	2.93322	2.53399	2.30600
	.02	6.96455	6.74650	6.11425	5.15113	4.08594	3.26852	2.89646
	.01	9.92482	9.60041	8.65326	7.16912	5.37322	3.92122	3.35538
8,3	.10	2.35336	2.32497	2.25566	2.16226	2.04690	1.92531	1.85955
	.05	3.18245	3.11985	2.95848	2.75753	2.55640	2.38349	2.30600
	.02	4.54070	4.42217	4.09465	3.66182	3.26606	2.99063	2.89646
	.01	5.84091	5.67135	5.18866	4.50575	3.86643	3.46675	3.35538
8,4	.10	2.13185	2.11734	2.08356	2.03412	1.96596	1.89365	1.85955
	.05	2.77645	2.73783	2.64571	2.53719	2.42813	2.33872	2.30600
	.02	3.74695	3.66846	3.46657	3.23183	3.03835	2.92235	2.89646
	.01	4.60409	4.49036	4.18357	3.80864	3.51656	3.37325	3.35538
8,5	.10	2.01505	2.00817	1.99304	1.96682	1.92467	1.87923	1.85955
	.05	2.57058	2.54551	2.49020	2.42830	2.36659	2.31979	2.30600
	.02	3.36493	3.30900	3.17584	3.03836	2.93940	2.89683	2.89646
	.01	4.03214	3.94880	3.73911	3.51764	3.37763	3.34202	3.35538
8,6	.10	1.94318	1.94107	1.93739	1.92565	1.90011	1.87130	1.85955
	.05	2.44691	2.43051	2.39783	2.36409	2.33142	2.30990	2.30600
	.02	3.14267	3.10137	3.01107	2.93059	2.88643	2.88455	2.89646
	.01	3.70742	3.64372	3.49556	3.36320	3.30751	3.32812	3.35538
8,7	.10	1.89458	1.89372	1.89977	1.89798	1.88400	1.86637	1.85955
	.05	2.36462	2.35424	2.33683	2.32201	2.30900	2.30398	2.30600
	.02	2.99795	2.96695	2.90576	2.86265	2.85424	2.87756	2.89646
	.01	3.49948	3.44964	3.34371	3.26908	3.26674	3.32058	3.35538
8,8	.10	1.85955	1.86305	1.87268	1.87816	1.87268	1.86305	1.85955
	.05	2.30600	2.30003	2.29361	2.29241	2.29361	2.30003	2.30600
	.02	2.89646	2.87309	2.83292	2.81624	2.83292	2.87309	2.89646
	.01	3.35538	3.31589	3.24063	3.20636	3.24063	3.31589	3.35538
8,10	.10	1.81246	1.81913	1.83628	1.85173	1.85791	1.85866	1.85955
	.05	2.22814	2.22818	2.23654	2.25371	2.27399	2.29518	2.30600
	.02	2.76377	2.75098	2.73905	2.75732	2.80676	2.86773	2.89646
	.01	3.16927	3.14423	3.11038	3.12882	3.20971	3.31038	3.35538
8,12	.10	1.78229	1.79099	1.81298	1.83496	1.84875	1.85633	1.85955
	.05	2.17881	2.18276	2.20060	2.22961	2.26210	2.29231	2.30600
	.02	2.68100	2.67519	2.68136	2.72173	2.79150	2.86462	2.89646
	.01	3.05454	3.03912	3.03189	3.08323	3.19228	3.30724	3.35538
8,24	.10	1.71088	1.72439	1.75798	1.79595	1.82813	1.85063	1.85955
	.05	2.06390	2.07727	2.11768	2.17507	2.23615	2.28615	2.30600
	.02	2.49216	2.50351	2.55261	2.64457	2.75976	2.85807	2.89646
	.01	2.79694	2.80587	2.86151	2.98813	3.15751	3.30068	3.35538
8,∞	.10	1.64481	1.66267	1.70724	1.76078	1.81019	1.84593	1.85955
	.05	1.95991	1.98224	2.04365	2.12801	2.21460	2.28080	2.30600
	.02	2.32631	2.35456	2.44366	2.58228	2.73495	2.85248	2.89646
	.01	2.57581	2.60869	2.72332	2.91568	3.13151	3.29509	3.35538
10,1	.10	6.31375	6.11207	5.52484	4.61261	3.82744	2.55628	1.81246
	.05	12.70620	12.28004	11.03227	9.05496	6.81053	3.83503	2.22814
	.02	31.82052	30.73898	27.56871	22.52832	15.96958	8.38856	2.76377
	.01	63.65674	61.48904	55.13402	45.02600	31.85787	16.54700	3.16927
10,2	.10	2.91999	2.85710	2.68798	2.46219	2.21731	1.97418	1.81246
	.05	4.30265	4.18361	3.84615	3.36571	2.87275	2.46025	2.22814
	.02	6.96455	6.74526	6.10803	5.12639	4.00853	3.14714	2.76377
	.01	9.92482	9.59954	8.64929	7.15506	5.30783	3.78881	3.16927
10,3	.10	2.35336	2.32149	2.24102	2.13421	2.00884	1.88118	1.81246
	.05	3.18245	3.11668	2.94213	2.71908	2.49828	2.31232	2.22814
	.02	4.54070	4.41971	4.08010	3.61400	3.17659	2.87184	2.76377
	.01	5.84091	5.66940	5.17731	4.48881	3.75252	3.50189	3.16927
10,4	.10	2.13185	2.11552	2.06873	2.00680	1.92885	1.84981	1.81246
	.05	2.77645	2.73396	2.62755	2.49844	2.37094	2.26755	2.22814
	.02	3.74695	3.66503	3.44664	3.17771	2.94756	2.80284	2.76377
	.01	4.60409	4.48737	4.16466	3.74491	3.39523	3.20891	3.16927

Table 2-continued

Percentage Points of the Behrens-Fisher Distribution for $\nu_1, \nu_2 = 1(1)8, 10, 12, 24, \infty$, $\theta = 0°(15°)90°$, $\alpha = .10, .05, .02, .01$

ν_1, ν_2	α	0°	15°	30°	45°	60°	75°	90°
10,5	.10	2.01505	2.00417	1.97820	1.93990	1.88796	1.83541	1.81246
	.05	2.57058	2.54123	2.47131	2.38957	2.30964	2.24842	2.22814
	.02	3.36493	3.30482	3.15304	2.98197	2.84767	2.77611	2.76377
	.01	4.03214	3.94495	3.71508	3.44726	3.25289	3.17280	3.16927
10,6	.10	1.94318	1.93698	1.92257	1.89897	1.86358	1.82745	1.81246
	.05	2.44691	2.42597	2.37857	2.32539	2.27449	2.23833	2.22814
	.02	3.14267	3.09667	2.98665	2.87300	2.79386	2.76320	2.76377
	.01	3.70742	3.63918	3.46829	3.28937	3.18042	3.15772	3.16927
10,7	.10	1.89458	1.89157	1.88498	1.87145	1.84786	1.82249	1.81246
	.05	2.36462	2.34952	2.31735	2.28331	2.25202	2.23223	2.22814
	.02	2.99795	2.96187	2.88031	2.80430	2.76096	2.75576	2.76377
	.01	3.49948	3.44456	3.31429	3.19308	3.13786	3.14942	3.16927
10,8	.10	1.85955	1.85886	1.85791	1.85173	1.83628	1.81913	1.81246
	.05	2.30600	2.29518	2.27399	2.25371	2.23654	2.22818	2.22814
	.02	2.89646	2.86773	2.80676	2.75732	2.73905	2.75098	2.76377
	.01	3.35538	3.31038	3.20971	3.12882	3.11038	3.14423	3.16927
10,10	.10	1.81246	1.81489	1.82155	1.82543	1.82155	1.81489	1.81246
	.05	2.22814	2.22316	2.21675	2.21497	2.21675	2.22316	2.22814
	.02	2.76377	2.74522	2.71201	2.69760	2.71201	2.74522	2.76377
	.01	3.16927	3.13811	3.07752	3.04917	3.07752	3.13811	3.16927
10,12	.10	1.78229	1.78672	1.79828	1.80873	1.81240	1.81233	1.81246
	.05	2.17881	2.17764	2.18072	2.19082	2.20472	2.22018	2.22814
	.02	2.68100	2.66916	2.65377	2.66146	2.69613	2.74187	2.76377
	.01	3.05454	3.03258	2.99783	3.00219	3.05882	3.13460	3.16927
10,24	.10	1.71088	1.72005	1.74334	1.76985	1.79174	1.80673	1.81246
	.05	2.06390	2.07191	2.09757	2.13609	2.17834	2.21377	2.22814
	.02	2.49216	2.49685	2.52379	2.58280	2.66283	2.73480	2.76377
	.01	2.79694	2.79797	2.82469	2.90352	3.02110	3.12730	3.16927
10,∞	.10	1.64481	1.65827	1.69265	1.73475	1.77372	1.80176	1.81246
	.05	1.95991	1.97666	2.02353	2.08872	2.15630	2.20823	2.22814
	.02	2.32631	2.34732	2.41370	2.51884	2.63659	2.72882	2.76377
	.01	2.57581	2.60007	2.68395	2.82748	2.99267	3.12118	3.16927
12,1	.10	6.31375	6.11152	5.52234	4.60428	3.50208	2.51828	1.78229
	.05	12.70620	12.27977	11.03111	9.05193	6.50145	3.78641	2.17881
	.02	31.82052	30.73887	27.56826	22.52720	15.96714	8.37694	2.68100
	.01	63.65674	61.48900	55.13380	45.02545	31.85669	16.54403	3.05454
12,2	.10	2.91999	2.85553	2.68020	2.44413	2.19122	1.94478	1.78229
	.05	4.30265	4.18245	3.84005	3.34591	2.83513	2.41353	2.17881
	.02	6.96455	6.74453	6.10457	5.11370	3.96417	3.07175	2.68100
	.01	9.92482	9.59902	8.64699	7.14800	5.27546	3.65982	3.05454
12,3	.10	2.35336	2.31943	2.23214	2.11654	1.98448	1.85288	1.78229
	.05	3.18245	3.11484	2.93265	2.69562	2.46174	2.26711	2.17881
	.02	4.54070	4.41828	4.07219	3.58675	3.12205	2.79772	2.68100
	.01	5.84091	5.66825	5.17128	4.43372	3.68559	3.20103	3.05454
12,4	.10	2.13185	2.11124	2.05963	1.98952	1.90509	1.82170	1.78229
	.05	2.77645	2.73170	2.61679	2.47452	2.33493	2.22254	2.17881
	.02	3.74695	3.66302	3.43550	3.14576	2.89185	2.72817	2.68100
	.01	4.60409	4.48564	4.15458	3.70911	3.32252	3.10356	3.05454
12,5	.10	2.01505	2.00179	1.96905	1.92285	1.86445	1.80732	1.78229
	.05	2.57058	2.53871	2.46000	2.36556	2.27378	2.20329	2.17881
	.02	3.36493	3.30241	3.14006	2.94825	2.79131	2.70120	2.68100
	.01	4.03214	3.94273	3.70200	3.40670	3.17787	3.06929	3.05454
12,6	.10	1.94318	1.93453	1.91341	1.88205	1.84019	1.79935	1.78229
	.05	2.44691	2.42329	2.36697	2.30134	2.23864	2.19308	2.17881
	.02	3.14267	3.09396	2.97257	2.83839	2.73697	2.68789	2.68100
	.01	3.70742	3.63658	3.45319	3.24635	3.10388	3.05345	3.05454
12,7	.10	1.89458	1.88908	1.87582	1.85462	1.82423	1.79437	1.78229
	.05	2.36462	2.34673	2.30557	2.25924	2.21613	2.18689	2.17881
	.02	2.99795	2.95893	2.86553	2.76912	2.70363	2.68017	2.68100
	.01	3.49948	3.44165	3.29783	3.14854	3.06022	3.04465	3.05454
12,8	.10	1.85955	1.85633	1.84875	1.83496	1.81298	1.79099	1.78229
	.05	2.30600	2.29231	2.26210	2.22961	2.20060	2.18276	2.17881
	.02	2.89646	2.86462	2.79150	2.72173	2.68136	2.67519	2.68100
	.01	3.35538	3.30724	3.19228	3.08323	3.03189	3.03912	3.05454
12,10	.10	1.81246	1.81233	1.81240	1.80873	1.79828	1.78672	1.78229
	.05	2.22814	2.22018	2.20472	2.19082	2.18072	2.17764	2.17881
	.02	2.76377	2.74187	2.69613	2.66146	2.65377	2.66916	2.68100
	.01	3.16927	3.13460	3.05882	3.00219	2.99783	3.03258	3.05454
12,12	.10	1.78229	1.78413	1.78913	1.79207	1.78913	1.78413	1.78229
	.05	2.17881	2.17459	2.16860	2.16664	2.16860	2.17459	2.17881
	.02	2.68100	2.66565	2.63751	2.62494	2.63751	2.66565	2.68100
	.01	3.05454	3.02883	2.97834	2.95429	2.97834	3.02883	3.05454
12,24	.10	1.71088	1.71742	1.73420	1.75325	1.76845	1.77848	1.78229
	.05	2.06390	2.06870	2.08526	2.11176	2.14195	2.16802	2.17881
	.02	2.49216	2.49295	2.50667	2.54532	2.60325	2.65823	2.68100
	.01	2.79694	2.79358	2.80338	2.85338	2.93875	3.02101	3.05454
12,∞	.10	1.64481	1.65560	1.68353	1.71819	1.75038	1.77346	1.78229
	.05	1.95991	1.97331	2.01105	2.06419	2.11963	2.16235	2.17881
	.02	2.32631	2.34305	2.39580	2.48034	2.57611	2.65198	2.68100
	.01	2.57581	2.59506	2.66098	2.77513	2.90874	3.01451	3.05454

Table 2-continued

Percentage Points of the Behrens-Fisher Distribution for $\nu_1, \nu_2 = 1(1)6, 10, 12, 24, \infty$, $\theta = 0°(15°)90°$, $\alpha = .10, .05, .02, .01$

ν_1, ν_2	α	0°	15°	30°	45°	60°	75°	90°
24,1	.10	6.31375	6.11033	5.51708	4.58825	3.44935	2.42872	1.71088
	.05	12.70620	12.27917	11.02860	9.04552	6.48488	3.68543	2.06390
	.02	31.82052	30.73864	27.56727	22.52476	15.96185	8.36187	2.49216
	.01	63.65674	61.48889	55.13331	45.02424	31.85410	16.53765	2.79694
24,2	.10	2.91999	2.85219	2.66385	2.40367	2.13004	1.87506	1.71088
	.05	4.30265	4.17997	3.82782	3.30555	2.75043	2.30468	2.06390
	.02	6.96455	6.74293	6.09740	5.09067	3.87742	2.90129	2.49216
	.01	9.92482	9.59789	8.64210	7.13443	5.22337	3.44555	2.79694
24,3	.10	2.35336	2.31502	2.21275	2.07602	1.92708	1.78580	1.71088
	.05	3.18245	3.11092	2.91307	2.64426	2.37778	2.16214	2.06390
	.02	4.54070	4.41523	4.05667	3.53290	3.00270	2.63011	2.49216
	.01	5.84091	5.66579	5.15940	4.38938	3.54801	2.97720	2.79694
24,4	.10	2.13185	2.10634	2.03948	1.94961	1.84905	1.75513	1.71088
	.05	2.77645	2.72690	2.59402	2.42117	2.25191	2.11789	2.06390
	.02	3.74695	3.65881	3.41338	3.07917	2.76817	2.55969	2.49216
	.01	4.60409	4.48197	4.13524	3.63976	3.16698	2.87643	2.79694
24,5	.10	2.01505	1.99662	1.94863	1.88333	1.80897	1.74082	1.71088
	.05	2.57058	2.53336	2.43573	2.31161	2.19104	2.09843	2.06390
	.02	3.36493	3.29734	3.11382	2.87652	2.66575	2.53186	2.49216
	.01	4.03214	3.93808	3.67670	3.32500	3.01587	2.83954	2.79694
24,6	.10	1.94318	1.92921	1.89288	1.84278	1.78498	1.73284	1.71088
	.05	2.44691	2.41758	2.34187	2.24711	2.15592	2.08798	2.06390
	.02	3.14267	3.08826	2.94374	2.76400	2.61009	2.51733	2.49216
	.01	3.70742	3.63114	3.42365	3.15805	2.93811	2.82192	2.79694
24,7	.10	1.89458	1.88365	1.85523	1.81550	1.76915	1.72782	1.71088
	.05	2.36462	2.34076	2.27997	2.20483	2.13333	2.08157	2.06390
	.02	2.99795	2.95275	2.83499	2.69309	2.57571	2.50897	2.49216
	.01	3.49948	3.43559	3.26527	3.05619	2.89181	2.81193	2.79694
24,8	.10	1.85955	1.85083	1.82813	1.79595	1.75798	1.72439	1.71088
	.05	2.30600	2.28615	2.23615	2.17507	2.11768	2.07727	2.06390
	.02	2.89646	2.85807	2.75976	2.64457	2.55261	2.50351	2.49216
	.01	3.35538	3.30068	3.15751	2.98813	2.86151	2.80557	2.79694
24,10	.10	1.81246	1.80673	1.79174	1.76985	1.74334	1.72005	1.71088
	.05	2.22814	2.21377	2.17834	2.13609	2.09757	2.07191	2.06390
	.02	2.76377	2.73480	2.66283	2.58280	2.52379	2.49685	2.49216
	.01	3.16927	3.12730	3.02110	2.90352	2.82469	2.79797	2.79694
24,12	.10	1.78229	1.77848	1.76845	1.75325	1.73420	1.71742	1.71088
	.05	2.17881	2.16802	2.14195	2.11176	2.08526	2.06870	2.06390
	.02	2.68100	2.65823	2.60325	2.54532	2.50667	2.49295	2.49216
	.01	3.05454	3.02101	2.93875	2.85338	2.80338	2.79358	2.79694
24,24	.10	1.71088	1.71163	1.71348	1.71455	1.71348	1.71163	1.71088
	.05	2.06390	2.06175	2.05803	2.05646	2.05803	2.06175	2.06390
	.02	2.49216	2.48466	2.47021	2.46331	2.47021	2.48466	2.49216
	.01	2.79694	2.78442	2.75946	2.74713	2.75946	2.78442	2.79694
24,∞	.10	1.64481	1.64970	1.66277	1.67953	1.69530	1.70651	1.71088
	.05	1.95991	1.96601	1.98331	2.00837	2.03504	2.05576	2.06390
	.02	2.32631	2.33392	2.35737	2.39592	2.44098	2.47773	2.49216
	.01	2.57581	2.58453	2.61313	2.66379	2.72567	2.77690	2.79694
∞,1	.10	6.31375	6.10602	5.50986	4.57412	3.40944	2.34652	1.64481
	.05	12.70620	12.27205	11.02059	9.03548	6.46939	3.61340	1.95991
	.02	31.82052	30.72189	27.55161	22.51063	15.94896	8.34623	2.32631
	.01	63.65674	61.45570	55.10323	44.99901	31.83484	16.52357	2.57581
∞,2	.10	2.91999	2.84858	2.65028	2.36919	2.07429	1.81026	1.64481
	.05	4.30265	4.17674	3.81739	3.27519	2.67858	2.20631	1.95991
	.02	6.96455	6.73976	6.09000	5.07317	3.81967	2.75461	2.32631
	.01	9.92482	9.59433	8.63577	7.12209	5.19258	3.27599	2.57581
∞,3	.10	2.35336	2.31086	2.19640	2.04023	1.87431	1.72353	1.64481
	.05	3.18245	3.10707	2.89742	2.60223	2.30375	2.06736	1.95991
	.02	4.54070	4.41186	4.04423	3.49449	2.90609	2.48453	2.32631
	.01	5.84091	5.66269	5.14929	4.35989	3.44907	2.78916	2.57581
∞,4	.10	2.13185	2.10187	2.02226	1.91399	1.79741	1.69336	1.64481
	.05	2.77645	2.72250	2.57562	2.37625	2.17823	2.02350	1.95991
	.02	3.74695	3.65480	3.39639	3.02862	2.66495	2.41347	2.32631
	.01	4.60409	4.47831	4.12041	3.59200	3.04526	2.68530	2.57581
∞,5	.10	2.01505	1.99197	1.93106	1.84790	1.75782	1.67915	1.64481
	.05	2.57058	2.52859	2.41590	2.26583	2.11746	2.00392	1.95991
	.02	3.36493	3.29278	3.09366	2.82034	2.56009	2.38438	2.32631
	.01	4.03214	3.93381	3.65784	3.26601	2.88632	2.64614	2.57581
∞,6	.10	1.94318	1.92444	1.87513	1.80747	1.73406	1.67117	1.64481
	.05	2.44691	2.41255	2.32122	2.20060	2.08229	1.99326	1.95991
	.02	3.14267	3.08327	2.92147	2.70481	2.50296	2.36939	2.32631
	.01	3.70742	3.62634	3.40171	3.09253	2.80447	2.62693	2.57581
∞,7	.10	1.89458	1.87881	1.83737	1.78028	1.71835	1.66612	1.64481
	.05	2.36462	2.33554	2.25878	2.16799	2.05960	1.98668	1.95991
	.02	2.99795	2.94742	2.81124	2.63204	2.46755	2.36045	2.32631
	.01	3.49948	3.43036	3.24102	2.98686	2.75852	2.61884	2.57581
∞,8	.10	1.85955	1.84593	1.81019	1.76078	1.70734	1.66267	1.64481
	.05	2.30600	2.28080	2.21460	2.12801	2.04388	1.98224	1.95991
	.02	2.89646	2.85248	2.73495	2.58235	2.44366	2.35456	2.32631
	.01	3.35538	3.29509	3.13181	2.91868	2.72332	2.60869	2.57581

BEST COPY AVAILABLE

Table 2-continued

Percentage Points of the Behrens-Fisher Distribution for $\nu_1, \nu_2 = 1(1)8, 10, 12, 24, \infty$, $\theta = 0°(15°)90°$, $\alpha = .10, .05, .02, .01$

ν_1, ν_2	α	0°	15°	30°	45°	60°	75°	90°
∞,10	.10	1.81246	1.80176	1.77372	1.73475	1.69265	1.65827	1.64481
	.05	2.22814	2.20823	2.15630	2.08872	2.02353	1.97666	1.95991
	.02	2.76377	2.72882	2.63659	2.51884	2.41370	2.34732	2.32631
	.01	3.16927	3.12118	2.99267	2.82748	2.68395	2.60007	2.57581
∞,12	.10	1.78229	1.77346	1.75038	1.71819	1.68353	1.65560	1.64481
	.05	2.17881	2.16235	2.11963	2.06419	2.01105	1.97331	1.95991
	.02	2.68100	2.65198	2.57611	2.48034	2.39580	2.34305	2.32631
	.01	3.05454	3.01451	2.90874	2.77513	2.66098	2.59506	2.57581
∞,24	.10	1.71088	1.70651	1.69530	1.67953	1.66277	1.64970	1.64481
	.05	2.06390	2.05576	2.03504	2.00837	1.98331	1.96601	1.95991
	.02	2.49216	2.47773	2.44098	2.39592	2.35737	2.33392	2.32631
	.01	2.79694	2.77690	2.72567	2.66379	2.61313	2.58453	2.57581
∞,∞	.10	1.64481	1.64481	1.64481	1.64481	1.64481	1.64481	1.64481
	.05	1.95991	1.95991	1.95991	1.95991	1.95991	1.95991	1.95991
	.02	2.32631	2.32631	2.32631	2.32631	2.32631	2.32631	2.32631
	.01	2.57581	2.57581	2.57581	2.57581	2.57581	2.57581	2.57581

Table 3
Critical Values and the 95% Confidence/HPD Intervals for
the Marascuilo and Serlin (1988) Data

Method	Critical Value	Interval
t^* Test	$4.30265 = t_{.025}(2)$	$[-18.97365, 9.14031]$
Bayesian/Fiducial	$4.13069 = \tau_{.025}(3, 2, 27.19365)$	$[-18.41183, 8.57850]$
Welch (order $1/\nu_i^2$)	$3.20703 = t^{\mathrm{W}}_{.025}(3, 2, 0.20885)$	$[-15.39420, 5.56086]$
Welch's Approximate t	$3.15117 = t_{.025}(3.05344)$	$[-15.21169, 5.37835]$
Student's t	$2.57058 = t_{.025}(5)$	$[-12.64209, 2.80875]$

Figure Captions

Figure 1. Integration of $p(\tau)$.

Figure 2. Posterior Distribution of δ.

Figure 1. Integration of $p(\tau)$.

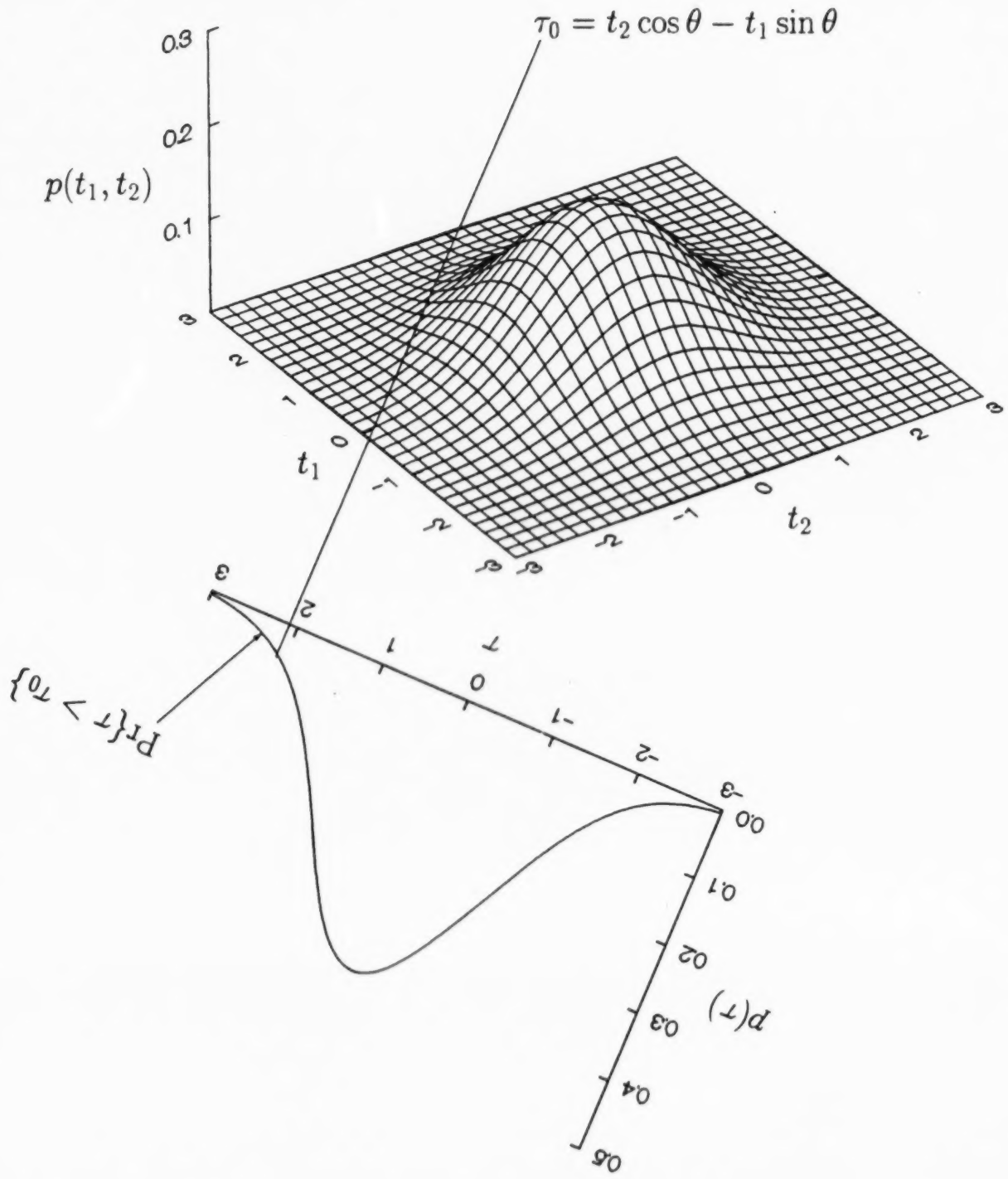

Figure 2. Posterior Distribution of δ.